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FX Options and smile risk

ANTONIO CASTAGNA

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Antonio Castagna

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Preface

When I first proposed writing a book on FX options, I could not help thinking that the final result would produce in the reader that disappointing, yet typically human, feeling caused by the recognition of what the Qoelet expresses in such a condensed way: "Quod factum est, ipsum est, quod faciendum est: nihil sub sole novum", which in slightly more modern words, and in accordance with the situation, means "Many books on options have been written in the past and this one is just telling the same old stories everybody knows". This fear was also sharpened by the fact that some very good books have already been written on the subject, so that just trying to be at the same level would be a titanic task. In this respect, I would like to mention here the excellent book by Uwe Wystup [63], which covers many areas, from pricing to regulation issues.

My scepticism about the likely outcome of my efforts was then partially reduced when, by chance, I read an aphorism of that solitary Colombian thinker (still inexplicably not too much known), Nicolas Davila, in his *Escolios a un texto implicito*, which stated: "Nobody thinks seriously until he cares about being original". I started to become aware that actually I did not have to search for new areas to analyse, and that I did not necessarily have to be original about the choice of subjects: "simply", I had to explore them deeply. Two questions naturally arose in my mind: Do I have the knowledge and expertise to undertake such a thorough inquiry? Besides, and probably more importantly, even if we assume that knowledge and expretise just for the sake of argument, why should I do it?

As far as the first question is concerned, I could not conceitedly say that my expertise derived from theoretical studies or technical skills, or from the fact that I was a smart trader capable of understanding the markets on any occasion, simply because none of that was true. Yet, in the year 2000, when I was working as a market maker on the interest derivatives (caps, floors and swaptions) market in Banca IMI, Milan, I was asked by the two heads of the dealing room to start a desk, market making in FX options. I had no experience in such a market, and nobody who could teach me about how it worked, or had ever worked, in the bank. So I began setting up pricing systems and risk management tools by relying only on my intuition and reasoning. Then, I started to make prices and manage the book, and so started to learn. I learnt in the only way living beings learn on earth, that is: by suffering. In the market-making context suffering means basically two things: losing money in its phenomenal aspect (which mainly concerns the financial institution) and feeling depressed in its psychological aspect (which mainly concerns the trader). Ultimately, I can say I achieved my expertise on FX options by suffering, so that I have no fear in claiming that my knowledge and understanding of FX options is not purely academic or theoretical, in which case I should admit my manifest inferiority to many people. Alternatively said, my knowledge is entirely due to the principle that the eighteenth-century philosopher Vico stated in his *Principi di Scienza Nuova*, according to which one really and fully knows something only if he has made it.

As far as the second question is concerned, it is relevant that in the year 2006 I stopped being in charge of the FX options desk in Banca IMI. I can safely say that (to use the scholastic philosopher's categories) if I was, in a more or less unconscious way, the efficient cause of the FX options desk, I was also, again in a more or less unconscious way, the final cause of it (at least in the way I liked it to operate). After two years I had stopped the market-making activity in FX options, but I did not want to forget and lose for ever all that I had assimilated during those six years. Writing a book is likely the best way to firmly fix all the concepts and the know-how that I absorbed from my experience.

As should be clear from all that has been said above, this book is written from a market-maker perspective and is focused mainly on problems related to pricing and risk management. I prefer to start with a list of what this book is not meant to be: it is not a mathematical finance textbook, although some basic options pricing theory will be presented and in general much mathematical formalism will be used; it is not aimed at showing all the possible structures that can be traded in the FX market, especially with a bank's customers (corporates, speculators, investors, etc.). Hence, I do not deal with aspects referring to the sell side. As a consequence of the previous point, I will not analyse all the possible existing kinds of contracts. Namely, I will not deal with Asian options, basket options and correlation contracts (range mountain options, for example). These options are typically used to build structured products for investors and they are very common in the equity options market. When currencies are considered as an asset class, then the same kind of options can have them as an underlying. Anyway, many books have been written on how to price such contracts, and how to manage their risk and, although they have their main reference to equities, their result can easily be extended to the FX market. In a few words, this book is not a collection of pricing formulae. Besides, I will not enter into details of the interest rates market and I will not examine how to build a discount factor curve by bootstrap procedures: I assume that we are already provided with discount factors for any maturity, even if I am aware that I am neglecting a very momentous subject, at least at the time of writing.

This book is aimed at examining all the relevant issues a market maker has to cope with, both in terms of pricing different kinds of contracts and managing their related risks. Many details, often overlooked in most textbooks or articles, will be examined explicitly. Actually, they represent the link between the theory and practice, and they have a dramatic impact on the profitability of an FX options desk. I will also provide many examples: since in most cases one must resort to numerical procedures, they will be described step-by-step and then worked out in practice.

After this preliminary warning, an overview of the outline of the book is in order. I will start, in the first chapter, with the basic definitions of the FX market: the definition of pairs and the description of the main contracts are presented. I will also illustrate the main conventions operating amongst professional market makers. The second chapter is devoted to a quick review of the main concepts of the option pricing theory and their application within a Black–Scholes (BS hereon) economy, and then a stochastic volatility environment. I introduce some models that could be implemented to price and manage FX options, although in subsequent chapters I will use only one of them as an example of the alternatives to the BS setting.

Managing the volatility risk is the main task of the options trader, so the entire third chapter is devoted to the effects of volatility on the profits and losses arising from the hedging activity. It is in this regard that the volatility smile is first introduced and examined. The fourth chapter extends the analysis to the building of a consistent volatility smile from a few options' market prices. Here I take the chance to remember that much of the work related to these topics has been conducted together with Fabio Mercurio, an exceptional colleague from the quantitative department in Banca IMI, and a good friend of mine too: it was a great intellectual pleasure to work with him and I thank him for sharing with me his experience and skills.

The fifth chapter dwells on the pricing of plain vanilla options and digital options, with much attention paid to some details and market conventions whose impact on the pricing is significant. In the sixth chapter barrier options are examined; they probably form the vast majority of the exotic options dealing in the FX market, so that they deserve an in-depth analysis and many tools and methods devised by practitioners will be described. By the same token, in the seventh chapter the other less common exotic options are examined.

The eighth chapter illustrates the tools for monitoring the main risks of an FX options book; besides, it shows and comments at some length on the behaviour, in terms of volatility risks, of the plain vanilla hedging instruments and of the main exotic options. The ninth and final chapter offers a quick analysis of the links among three currencies, and sketches an extension of the methods examined in the previous chapters to the contracts depending on many pairs.

One noteworthy feature of most of the methods and approaches described is that they hinge mainly on the BS model, which is still the main working tool in the market, although its flaws have been identified and discussed abundantly during the last 30 years. The reason for the striking inconsistency between the ascertained deficiencies of the BS model, and its widespread use in the FX market, is not due to the fact that market makers are stupidly stubborn (or, at least, they are not completely stupidly stubborn): on the contrary, they are aware of the risks that the model is not able to consider and include them in the pricing by resorting to sophisticated, yet definitely empirical (mis-)uses of the model, sometimes designed in a very clever way, even if from a theoretical perspective the adopted solutions may make academicians turn their noses up. I would like to define this as a "Dionysian" approach to the problems related to FX options: the complexity and even the inconsistency of the real world is accepted and faced with all the means we have at our disposal, although a reasonable rigour is needed in the choice of them. In contrast, I would see an "Apollonian" approach as aimed at the perfection of the formal theory, at the elegance of the derivation of the results and the beauty of the internal consistency of the models: the fascination for all of these is manifestly congenital to human nature (at least the most noble part of it) but, alas, they are not enough to account for all the noxious details of the real world. As usually happens, a combination of the two approaches, an "Apollonian" vision of a "Dionysian" experience, as someone wrote somewhere, is likely to produce the best results. I believe this is what actually occurs in the FX options market (and in other markets too, to be honest). On the other hand, if they say that options trading is an art, then FX options trading is the *Oedipus Rex*, or the Sistine Chapel if you prefer visual works.

I do not mean to start from the origin of the universe to thank all the people and events that made possible the writing of this book, but I cannot help mentioning my parents, who wanted me to study at LUISS University in Rome; there I took a degree in Financial Markets' Economics, under the supervision of Professor Emilio Barone, with a thesis on the pricing of American options. Professor Barone, whose bright mind I admire, was the first to encourage my studies in finance and I was honoured to write with him two articles. I would like to thank all the people who worked with me on the FX options desk in Banca IMI, even if for a short time: Roberto Binello, Marek Fogiel, Giuseppe Levato, Michele Lanza (who succeeded me as the head of the desk and who contributed greatly to its development), Andrej Mariani, Cristina Castagner and Alessandro Gavazzeni. I would also like to mention my colleagues and friends from the interest rate options desk: Luca Dominici, Stefano De Nuccio, Pierluigi D'Orazio and Davide Moresco. In the same bank I had the lucky chance to work in a stimulating environment with an exceptional quantitative department: besides the already mentioned Fabio Mercurio, I had interesting discussions with Francesco Rapisarda, Andrea Bugin, Damiano Brigo, Giulio Sartorelli and Lorenzo Bisesti. I have to acknowledge also the illuminating talks that I had with my colleagues and friends Cristiano Cosso, Francesco Fede, Raffaele Giura and Sergio Grasso.

Paola Mosconi deserves special thanks for proofreading the manuscript and for suggesting many improvements. The suggestions of anonymous reviewers are greatly acknowledged as well.

Although not directly related to the ideas and concepts discussed in this book, still all my friends in Milan (many of whom I have known since I was at the university) had a more or less hidden role: I would like to thank them for all their support and affection.

Finally, I must thank the last two top managers I had as my bosses in Banca IMI: Andrea Crovetto and Gianluca Cugno, whose decisions, unconsciously and unwittingly according to the utmost perfect heterogenesis of ends, ultimately allowed me to write this book.

Notation and Acronyms

- S_t : spot price of the exchange rate at time t
- $F(t, T)$: forward price of the exchange rate at time t for a contract expiring at time T
- $r^d(t)$,r_t^d : domestic spot rate at time t. It may be continuous, simple or annual compounded according to the context
- $r^f(t)$,r_t^f : foreign spot rate at time t. It may be continuous, simple or annual compounded according to the context
- $P^d(t, T) = E^Q[e^{-\int_t^T r^d(s)ds}]$: domestic zero-coupon bond price expiring at time T prevailing at time t
- $P^f(t, T) = E^Q[e^{-\int_t^T r^f(s)ds}]$: foreign zero-coupon bond price expiring at time T prevailing at time t
- $D^d(t) = D_t^d = e^{\int_0^t r^d(s)ds}$: domestic deposit (bank account) accruing interest at the domestic rate r^d with initial value in domestic currency units $D^d(0) = 1$
- $D^f(t) = D_t^f = e^{\int_0^t r^f(s)ds}$: foreign deposit (bank account) accruing interest at the foreign rate r^f with initial value in foreign currency units $D^f(0) = 1$
- H_t : barrier level at time t
- τ : time between t and T expressed as a year fraction, i.e. $\tau = \frac{T-t}{365}$
- $T_1, T_2, ..., T_i - 1, T_i$: set of maturities
- ς_t : instantaneous volatility of exchange rate spot process at time t
- $\sigma(K, T), \sigma(K)$: implied volatility to plug into the Bl formula for an option struck at K and expiring in T
- Q : risk-neutral measure
- Q^T : forward risk-adjusted measure (the domestic zero-coupon $P(t, T)$ is the numeraire)
- $E[x]$: expected value of x under the physical measure
- $E^Q[x]$: expected value of x under the risk-neutral measure
- $E^T[x]$: expected value of x under the forward risk-adjusted measure
- $\mathcal{N}(\mu, \sigma)$: normal distribution with mean μ and variance σ
- $\Phi(x)$: cumulative distribution function of a standard Gaussian distribution calculated in x
- W_t, Z_t : Brownian motions under the real-world measure
- W_t^Q, Z_t^Q : Brownian motions under the risk-neutral measure
- $\mathcal{O}(\cdot)$: price of a European contingent claim, such as a plain vanilla European option
- $\mathrm{Bl}(S_t, t, T, K, P^d(t, T), P^f(t, T), \sigma, \omega)$: price of a plain vanilla European option at time t and expiring at time T, struck at K and evaluated according to the BS model with a forward price of the exchange rate $F(t; T)$, an implied volatility equal to σ and with the price of the

domestic zero-coupon bond equal to $P^d(t, T)$. If the option is a call then $\omega = 1$, if it is a put then $\omega = -1$

- **C**$(\cdot)$: price of a plain vanilla European call option. The function's arguments vary according to the context
- **P**$(\cdot)$: price of a plain vanilla European put. The function's arguments vary according to the context
- p : an option's premium
- $\mathcal{E}(\cdot)$: price of a generic exotic option
- $\mathcal{B}(\cdot)$: price of a generic European barrier option, such as an up&out call option
- $\mathcal{D}B(\cdot)$: price of a generic European double-barrier option
- **KOC** : price of a knock-out call option
- **KOP** : price of a knock-out put option
- **KIC** : price of a knock-in call option
- **KIP** : price of a knock-in put option
- **UOC** : price of an up&out call option
- **DOC** : price of a down&out call option
- **UIC** : price of an up&in call option
- **DIC** : price of a down&in call option
- **UOP** : price of an up&out put option
- **DOP** : price of a down&out put option
- **UIP** : price of an up&in put option
- **DIP** : price of a down&in put option
- **OTH** : price of a one-touch option whose nominal amount is paid at the hit of the barrier level
- **OTE** : price of a one-touch option whose nominal amount is paid at the expiry of the contract
- **NT** : price of a no-touch option
- **DKOC** : price of a double-knock-out call option
- **DKOP** : price of a double-knock-out put option
- **DKIC** : price of a double-knock-in call option
- **DKIP** : price of a double-knock-in put option
- **DNT** : price of a double-no-touch option
- **DTE** : price of a double-touch option, paid at expiry
- **Fw**(t, T) : value of a forward contract (outright) at time t, expiring at time T
- **Fsw**(t, T) : value of an FX swap contract at time t, expiring at time T
- **STDL** : ATM straddle, i.e. a trading strategy (structure) involving the buying of a call and of a put struck at the same ATM level
- **RR** : risk reversal, i.e. a trading strategy (structure) involving the buying of a call against the selling of a put
- **VWB** : Vega-weighted butterfly, i.e. a trading strategy (structure) involving the buying of a strangle against the selling of an ATM straddle in such an amount as to make the total (BS model) Vega position nil
- **stdl** : ATM straddle price, in terms of BS implied volatility
- **RR** : risk reversal, i.e. a trading strategy (structure) involving the buying of a call against the selling of a put
- **rr** : risk reversal price, in terms of BS implied volatility

- **VWB** : Vega-weighted butterfly, i.e. a trading strategy (structure) involving the buying of a strangle against the selling of an ATM straddle in such an amount as to make the total (BS model) Vega position nil
- **vwb** : Vega-weighted butterfly price, in terms BS of implied volatility
- ATM : at-the-money level of the strike price of an option
- OTM : out-of-the-money level of the strike price of an option
- ITM : in-the-money level of the strike price of an option
- SDE : stochastic differential equation
- PDE : partial differential equation
- BS : Black–Scholes
- SV : stochastic volatility
- UV : uncertain volatility
- MIX : lognormal mixture

1
The FX Market

The foreign exchange (FX) market is an OTC market where each participant trades directly with the others; there is no exchange, though we can identify some major geographic trading centres: London (the primary centre, where the primary banks' market makers are located; its importance has increased in the last few years), New York, Tokyo, Singapore and Sydney. This means that trading activity is carried out 24 hours a day, though in practice during London working hours the market has the most liquidity. Needless to say, the FX market experiences fierce competition amongst participants.

Most trades are currently carried out via interbank platforms (EBS is the most important). Anyway, the major market makers offer Internet platforms to their clients for quick trades and for leaving orders. The Reuters Dealing, which was the main platform in the past, has lately lost much of its pre-eminence. Basically, it is a chat system connecting the participants, capable of recognizing the deal implicit in typical conversations between two professional operators, and transforming it into an automatic confirmation for the transaction. Nowadays, the Reuters Dealing is used mainly by option traders.

1.1 FX RATES AND SPOT CONTRACTS

Definition 1.1.1. ***FX rate***. *An exchange (FX) rate is the price of one currency in terms of another currency; the two currencies make a* ***pair***. *The* ***pair*** *is denoted by a label, made up of two tags of three characters each: each currency is identified by its tag. The first tag in the exchange rate is the base* ***currency***, *the second is the* ***numeraire currency***. *So the FX is the price of the base currency in terms of the numeraire currency.*

The numeraire currency can be considered as domestic: actually, in what follows we will refer to it as domestic. The base currency can be regarded as an asset whose trading generates profits and/or losses in terms of the domestic currency. In what follows the base currency will also be referred to as the foreign currency. We would like to stress that these denominations are not related to the perspective of the trader, who can actually be located anywhere and for whom the foreign currency may turn out to be indeed the domestic currency, from a "civil" point of view.

Example 1.1.1. *The euro/US dollar FX rate is identified by the label EURUSD and it denotes how many US dollars are worth 1 euro. The domestic (numeraire) currency is the US dollar and the foreign (base) currency is the euro.*

For each currency specific market conventions apply, and two of them are also important for the FX market: the *settlement date* and the *day count*. The settlement date (or delivery date) is the number of business days needed to actually transfer funds (if any are due) amongst interbank market participants after the closing of a deal; for most currencies it is two business days, but there are exceptions. In the market lore it is commonly referred to as "T + *number of days*", where "T" stands for the time (day) when the deal is closed. The *day count* is the

Table 1.1 *Settlement date* and *day count* conventions for some major currencies

Tag	Currency	Settlement (T +)	Day count
AUD	Australian dollar	2	act/360
CAD	Canadian dollar	2	act/360
CHF	Swiss franc	2	act/360
CZK	Czech koruna	2	act/360
DKK	Danish krone	2	act/360
EUR	Euro	2	act/360
GBP	UK pound	0	act/365
HKD	Hong Kong dollar	2	act/365
JPY	Japanese yen	2	act/360
NOK	Norwegian kroner	2	act/360
NZD	New Zealand dollar	2	act/360
PLN	Polish zloty	2	act/360
SEK	Swedish krona	2	act/360
USD	US dollar	2	act/360
ZAR	South African rand	2	act/365

time factor used to calculate accrued interest between two dates in the money market of the relevant currency; it usually applies for simple compounding. A list of some currencies and their related settlement date and day count conventions is given in Table 1.1.

The settlement date and the day count for each currency are useful to price forward (outright) and FX swap contracts. There is a settlement date specific for the spot contract though, and it is the number of days, after the trade date, when the two amounts denominated in the currencies involved are exchanged between the counterparties. The rules to determine the settlement date for a spot contract are a little more complex, since they need the intersection of three calendars: we list them below when we define the spot contract.

The FX rates are expressed as five-digit numbers, with no regard for the number of decimals; the fifth digit is named *pip*: 100 pips make a *figure*. As an example, the major FX rates for spot contracts (we will define *spot* below) as of 29 October 2007 are shown in Figure 1.1. Regular trades are for fixed amounts of the base currency. For example, if a trader asks for a spot price via the Reuters Dealing in the EURUSD, and they write

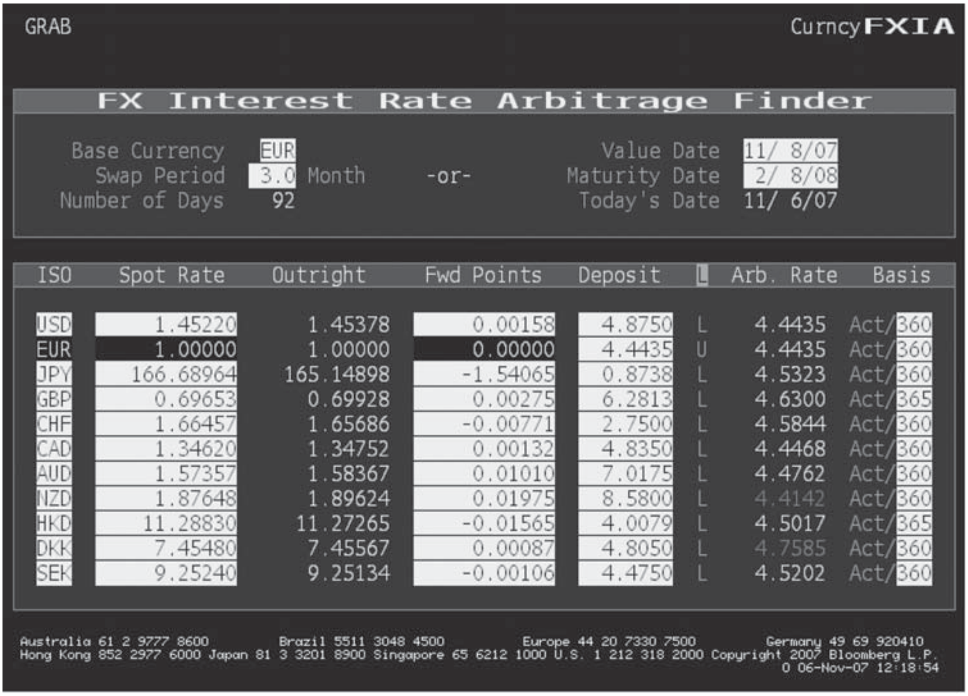
"I Buy (or Sell) 2 mios EURUSD at 1.3597"

this means that the trader buys (or sells) 2 million euros against 2 719 400 US dollars (1.3597 $\times$ 2 mios). Clearly, should one need exactly 1 million US dollars, it has to be specified as follows:

"I Buy 1 mio USD against EUR at 1.3597"

This means that the trader buys 1 million US dollars against 735 456 euros (1/1.3597 $\times$ 1 million). The two contracts closed in the examples are *spot* and the employed FX rate is also said to be *spot*. We define the spot contract as follows:

Definition 1.1.2. ***Spot***. *Two counterparties entering into a spot contract agree to exchange the base currency amounts against an amount of the numeraire currency equal to the spot FX rate. The settlement date is usually two business days after the transaction date (but it depends on the currency).*



GRAB CurncyFXIA

FX Interest Rate Arbitrage Finder

Base Currency EUR
Swap Period 3.0 Month -or- Maturity Date 2/ 8/08
Number of Days 92
Value Date 11/ 8/07
Today's Date 11/ 6/07

ISO	Spot Rate	Outright	Fwd Points	Deposit	L	Arb. Rate	Basis
USD	1.45220	1.45378	0.00158	4.8750	L	4.4435	Act/360
EUR	1.00000	1.00000	0.00000	4.4435	U	4.4435	Act/360
JPY	166.68964	165.14898	-1.54065	0.8738	L	4.5323	Act/360
GBP	0.69653	0.69928	0.00275	6.2813	L	4.6300	Act/365
CHF	1.66457	1.65686	-0.00771	2.7500	L	4.5844	Act/360
CAD	1.34620	1.34752	0.00132	4.8350	L	4.4468	Act/360
AUD	1.57357	1.58367	0.01010	7.0175	L	4.4762	Act/360
NZD	1.87648	1.89624	0.01975	8.5800	L	4.4142	Act/360
HKD	11.28830	11.27265	-0.01565	4.0079	L	4.5017	Act/365
DKK	7.45480	7.45567	0.00087	4.8050	L	4.7585	Act/360
SEK	9.25240	9.25134	-0.00106	4.4750	L	4.5202	Act/360

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2007 Bloomberg L.P.
0 06-Nov-07 12:18:54

Source: Bloomberg.

Figure 1.1 FX rates as of 29 October 2007 (Reproduced with permission)

As mentioned above, the settlement date for a spot contract is set according to specific rules involving three calendars (collapsing to two if the US dollar is one of the currencies of the traded pair). Here they are:

1. As a general rule, the settlement date for a spot contract is two business days after the trade date (T + 2), if this date is a business day for each of the two currencies of the pair. If this is not the case, the date is shifted forward until the condition is matched. An exception to this rule is the USDCAD (i.e., the US dollar/Canadian dollar pair), for which the settlement date is one business day after the trade date.
2. The settlement date set as in (1) must also be a business day in the USA, otherwise the date is shifted one day forward and the condition that the new date is a business day for each currency has to be checked again.
3. When the date after the trade date is a holiday in the USA (except for weekends), but not in other countries, then this date is counted as a business day to determine the settlement date. In this case it happens that for two days spot contracts will be settled on the same date, and in the market lore we say that the "settlement date is repeated".

We provide an example to clarify how to actually apply these rules.

Example 1.1.2. *Assume we are on Tuesday* 20 *November* 2007*; from market calendars it can be seen that Thursday* 22 *November is a holiday in the USA and Friday* 23 *November is a holiday in Japan. Consider three currencies: the US dollar, the euro and the yen. We consider the following possible trades with the corresponding settlement dates:*

- *On* 20 *November we close a spot contract in EURUSD. The settlement date will be* 23 *November: two business days would imply* 22 *November, but this is a holiday in the USA, so the settlement date is shifted forward one day, a "good" business day for both currencies.*
- *On* 21 *November we close a spot contract in EURUSD. The settlement date will be* 23 *November (repeated): the holiday in the USA is one day after the trade and is not a weekend, so it is taken as a business day.*
- *On* 20 *November we close a spot contract in USDJPY. The settlement date will be* 26 *November:* 22 *November is a holiday in the USA, so the settlement date is shifted forward one day, but* 23 *November is a holiday in Japan, so the settlement date is shifted forward to the first available business day, which is Monday* 26 *November, after the weekend. The same calculation also applies if we traded in EURJPY.*
- *On* 21 *November we close a spot contract in USDJPY. The settlement date will be* 26 *November:* 22 *November is a holiday in the USA but it is taken as a business day; anyway,* 23 *November is a holiday in Japan but it is not counted as a business day, so the settlement date is shifted forward to the first available business day, which is Monday* 26 *November, after the weekend.*
- *On* 22 *November we close a spot contract in EURUSD; it is a US holiday but we can trade in other countries. The settlement date will be* 26 *November:* 23 *November is a "good" business day for both currencies, then there is the weekend, and Monday* 26 *November is the second business day.*
- *On* 22 *November we close a spot contract in EURJPY. The settlement date will be* 27 *November:* 23 *November is a good business day for the euro, but not for the yen, so we skip after the weekend, and Tuesday* 27 *November is the second business day, "good" for both currencies and the US dollar as well.*

The rules for the calculation of the settlement date are probably the only real market-related technical issue a trader has to know, then they are ready to take part in the fastest game in town.

1.2 OUTRIGHT AND FX SWAP CONTRACTS

Outright (or forward) contracts are a simple extension of a spot contract, as is manifest from the following definition:

Definition 1.2.1. ***Outright***. *Two counterparties entering into an outright (or forward) contract agree to exchange, at a given expiry (settlement) date, the base currency amounts against an amount of the numeraire currency equal to the (forward) exchange rate.*

It is quite easy to see that the outright contract differs from a spot contract only for the settlement date, which is shifted forward in time up to the expiry date in the future. That, however, also implies an FX rate, which the transaction is executed at, different from the spot rate and the problem of its calculation arises. Actually, the calculation of the forward FX price can easily be tackled by means of the following arbitrage strategy:

Strategy 1.2.1. *Assume that we have an XXXYYY pair and that the spot FX rate is S_t at time t, whereas $F(t, T)$ is the forward FX rate for the expiry at time T. At time t, we operate the following:*

- *Borrow one unit of foreign currency XXX.*

- *Change one unit of XXX (foreign) against YYY and receive S_t YYY (domestic) units.*
- *Invest S_t YYY in a domestic deposit.*
- *Close an outright contract to change the terminal amount back into XXX, so that we receive $S_t \frac{1}{P^d(t,T)} \frac{1}{F(t,T)}$ XXX.*
- *Pay back the loan of one YYY plus interest.*

To avoid arbitrage, the final amount $S_t \frac{1}{P^d(t,T)} \frac{1}{F(t,T)}$ XXX must be equal to the value of the loan of 1 XXX at time T, which can be calculated by adding interest to the notional amount.

This strategy can be translated into formal terms as:

$$S_t \frac{1}{P^d(t, T)} \frac{1}{F(t, T)} = 1 \frac{1}{P^f(t, T)}$$

which means that we invest the S_t YYY units in a deposit traded in the domestic money market, yielding at the end $S_t \frac{1}{P^d(t,T)}$ ($P^d(t, T)$ is the price of the domestic pure zero-coupon bond), and change then back to XXX currency at the $F(t, T)$ forward rate. This has to be equal to 1 XXX units plus the interest prevailing in the foreign money market ($P^f(t, T)$ is the price of the foreign pure zero-coupon bond). Hence:

$$F(t, T) = S_t \frac{P^f(t, T)}{P^d(t, T)} \tag{1.1}$$

In Chapter 2 we will see an alternative, and more thorough, derivation for the fair price of a forward contract. The FX rate in equation (1.1) is that which makes the value of the outright contract nil at inception, as it has to be since no cash flow from either party is due when the deal is closed.

A strategy can also be operated by borrowing money in the domestic currency, investing it in a foreign deposit and converting it back into domestic currency units by an outright contract. It is easy to see that we come up with the same value of the fair forward price as in equation (1.1), which prevents any arbitrage opportunity.

The careful reader has surely noticed that in Strategy 1.2.1 the prices of pure discount bonds have been used to calculate the present and future value of a given currency amount. Actually, the market practice is to use money market conventions to price the deposits and hence to determine the forward FX rates. The use of pure discount bonds (also known as discount factors) is perfectly consistent with the market methodology as long as they are derived by a *bootstrap* procedure from the available market prices of the deposits.

Remark 1.2.1. *Strategy 1.2.1 is model-independent and operating it carries the forward price $F(t, T)$ at a level consistent with the other market variables (*i.e., *the FX spot rate and the domestic and foreign interest rates), so any arbitrage opportunity is cleared out. It should be stressed that two main assumptions underpin the strategy: (i) counterparties are not subject to default risk, and (ii) there is no limit to borrowing in the money markets.*

Assume that the first assumption does not hold. When we invest the amount denominated in YYY in a deposit yielding domestic interest, we are no longer sure of receiving the amount $S_t \frac{1}{P^d(t,T)}$ at time T to convert back into XXX units since the counterparty, to whom we lent money, may go bankrupt. We could expect to recover a fraction of the notional amount of the deposit, but the strategy is no longer effective anyway. In this case we may have a forward price

$F(t, T)$ trading in the market which is different from that determined univocally by Strategy 1.2.1, and we cannot operate the latter to exploit an arbitrage opportunity, since we would bear a risk of default that is not considered at all.

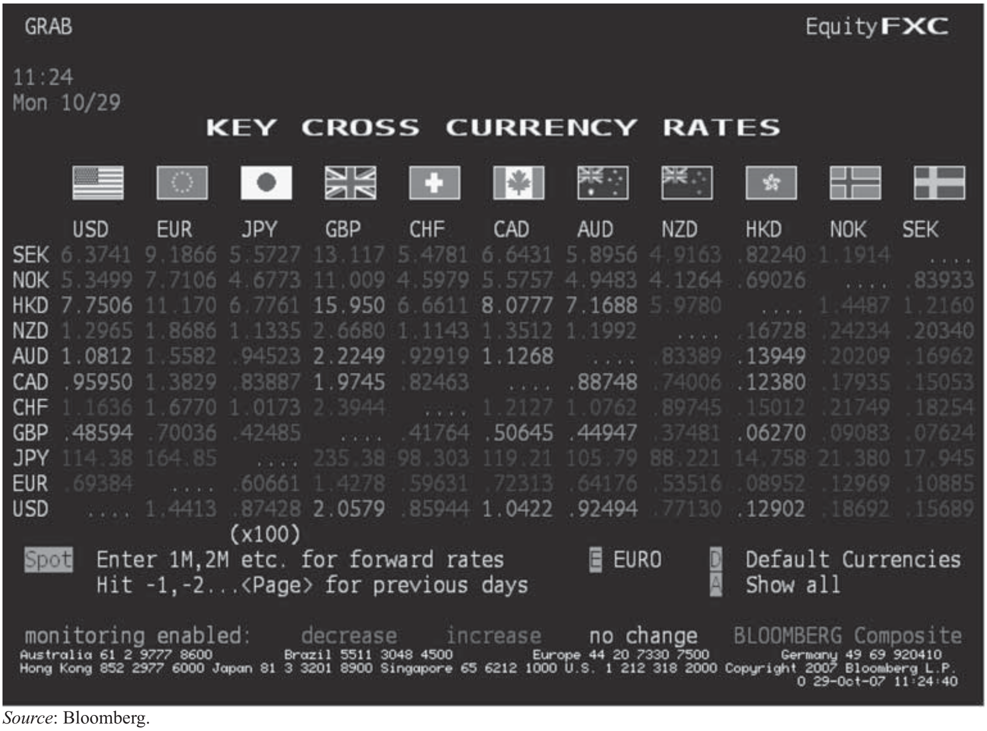
Assume now that the second assumption does not hold. We could observe a forward price in the market higher than that determined by Strategy 1.2.1, but we are not able to exploit the arbitrage opportunity just because there is a limited amount of lending in the market, so we cannot borrow the amount of one unit of XXX currency to start the strategy.

In reality, both situations can be experienced in the market and actually the risk of default can also strongly affect the amount of money that market operators are willing to lend amongst themselves. Starting from July 2007, a financial environment with a perceived high default risk related to financial institutions and a severe shrinking of the available liquidity has been very common, so that arbitrage opportunities can no longer be fully cleared out by operating the replication Strategy 1.2.1.

In the market, outright contracts are quoted in forward points:

$$\mathbf{Fpts}(t, T) = F(t, T) - S_t$$

Forward points are positive or negative, depending on the interest rate differentials, and they are also a function of the level of the spot rate. They are (algebraically) added to the spot rate when an outright is traded, so as to get the fair forward FX rate. In Figure 1.2, forward points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Mon 10/29

KEY CROSS CURRENCY RATES

	USD	EUR	JPY	GBP	CHF	CAD	AUD	NZD	HKD	NOK	SEK
SEK	6.3741	9.1866	5.5727	13.117	5.4781	6.6431	5.8956	4.9163	.82240	1.1914	....
NOK	5.3499	7.7106	4.6773	11.009	4.5979	5.5757	4.9483	4.1264	.69026	....	.83933
HKD	7.7506	11.170	6.7761	15.950	6.6611	8.0777	7.1688	5.9780	....	1.4487	1.2160
NZD	1.2965	1.8686	1.1335	2.6680	1.1143	1.3512	1.1992	....	.16728	.24234	.20340
AUD	1.0812	1.5582	.94523	2.2249	.92919	1.1268	....	.83389	.13949	.20209	.16962
CAD	.95950	1.3829	.83887	1.9745	.82463	....	.88748	.74006	.12380	.17935	.15053
CHF	1.1636	1.6770	1.0173	2.3944	....	1.2127	1.0762	.89745	.15012	.21749	.18254
GBP	.48594	.70036	.42485	....	.41764	.50645	.44947	.37481	.06270	.09083	.07624
JPY	114.38	164.85	....	235.38	98.303	119.21	105.79	88.221	14.758	21.380	17.945
EUR	.69384	....	.60661	1.4278	.59631	.72313	.64176	.53516	.08952	.12969	.10885
USD	....	1.4413	.87428	2.0579	.85944	1.0422	.92494	.77130	.12902	.18692	.15689

(x100)

Spot Enter 1M,2M etc. for forward rates E EURO D Default Currencies
Hit -1,-2...<Page> for previous days A Show all

monitoring enabled: decrease increase no change BLOOMBERG Composite
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2007 Bloomberg L.P.
0 29-Oct-07 11:24:40

Source: Bloomberg.

Figure 1.2 Forward points at 6 November 2007 (Reproduced with permission) at 6 November 2007 for a three-month delivery are shown – they are the same points used in FX swap contracts, which will be defined below. The base currency is the euro and forward points are referred to each (numeraire) currency listed against the euro: in the column "Arb. rate" the forward implied no-arbitrage rate for the euro is provided and it is derived from the formula to calculate the forward FX rate so as to match the market level of the latter.

For the sake of clarity and to show how forward FX rates are actually calculated, we provide the following example:

Example 1.2.1. *Assume we have the market data as in Figure* 1.2. *We want to check how the forward points for the EURUSD are calculated. We use formula* (1.1) *to calculate the forward FX rate, but we apply the money market conventions for capitalization and for discounting (i.e., simple compounding):*

$$F(0, 3M) = 1.4522 \frac{\left(1 + 4.875\% \frac{92}{360}\right)}{\left(1 + 4.4435\% \frac{92}{360}\right)} = 1.45378$$

where $3M$ *stands for "three-month expiry". Hence, the FX swap points are calculated straightforwardly as:*

$$\mathbf{Fpts}(0, 3M) = F(0, 3M) - S_0 = 1.45378 - 1.4522 = 0.00158$$

so that both the forward FX rate and forward points are verified by what is shown in the figure.

The FX swap is a very popular contract involving a spot and an outright contract:

Definition 1.2.2. ***FX swap.*** *Two counterparties entering into an FX swap contract agree to close a spot deal for a given amount of the base currency, and at the same time they agree to reverse the trade by an outright (forward) with the same base currency amount at a given expiry.*

From the definition of an FX swap, the valuation is straightforward: it is the sum of a spot contract and the value of a forward contract. So, we just need the spot rate and the forward points, which are denominated (FX) swap points when referred to such a contract. A typical request by a trader on the Reuters Dealing (which is still one of the main platforms where FX swap contracts can be traded) might be:

"I buy and sell back 1 mio EUR against USD in 3 months"

This means that the trader enters into a spot contract buying 1 million euros against US dollars, and then sells them back at the expiry of the FX swap in three months' time. We use market data provided in the Bloomberg screen shown in Figure 1.2 to see, in practice, how the FX swap contract implied by the request above is quoted and traded. Besides, in the example the difference between a *par* (alternatively an *even*) FX swap and a *non-par* (alternatively an *uneven* or *split* or *change*) FX swap is stressed.

Example 1.2.2. *We use the same market data as in Example* 1.2.1 *and in Figure* 1.2. *The current value of a* 3*M FX swap "buy and sell back 1 mio EUR against USD" has to be split* *into its domestic (US dollar in our case) and foreign (euro) components:*

$$\mathbf{Fsw}^d(0, 3M) = -S_0 + \frac{1}{(1 + r^d\tau)}F(0, 3M)$$
$$= -1.4522 + 1.45378\frac{1}{\left(1 + 4.875\%\frac{92}{360}\right)} = -0.0163 \text{ USD}$$
$$\mathbf{Fsw}^f(0, 3M) = 1 - \frac{1}{(1 + r^f\tau)}$$
$$= 1 - \frac{1}{\left(1 + 4.4435\%\frac{92}{360}\right)} = 0.0112 \text{ EUR}$$

In the two formulae above we just calculated the present value for all the cash flows provided by the FX swap contract, separately for each of the two currencies involved. An outflow of S_0 *US dollars against 1 euro at inception and an inflow of* $F(0, 3M)$ *on the delivery date against 1 euro again. The two final values are expressed for each leg of the corresponding currency. This is a par FX swap contract, since the notional amount (1 million euros) exchanged at inception via the spot transaction, and the final amount exchanged back at expiry, via the outright transaction, are the same. It is manifest that a par FX swap engenders a position different from* 0 *in both currencies. Professional market participants prefer to have nil currency exposure (we will see why later), so they prefer to trade non-par FX swaps. In this trade the amount of the base currency exchanged at the forward expiry is modified so as to generate a zero currency exposure. It is easy to see that the amount to be exchanged (so as to have a par FX swap) has to be compounded at the numeraire (foreign) currency interest rate. Hence, if we set the amount of euros to be exchanged on the delivery date equal to* $(1 + 4.4435\%\frac{92}{360}) = 1.0114$ *instead of* 1, *we get:*

$$\mathbf{Fsw}^d(0, 3M) = -S_0 + \frac{(1 + r^f\tau)}{(1 + r^d\tau)}F(0, 3M)$$
$$= -1.4522 + 1.45378\frac{\left(1 + 4.4435\%\frac{92}{360}\right)}{\left(1 + 4.875\%\frac{92}{360}\right)} = 0 \text{ USD}$$
$$\mathbf{Fsw}^f(0, 3M) = 1 - \frac{(1 + r^f\tau)}{(1 + r^f\tau)}$$
$$= 1 - 1 = 0 \text{ EUR}$$

which clearly shows no residual exposure to the FX risk.

The quoted price of an FX swap contract will be simply the forward points. They are related to the FX spot level, to be specified when closing the contract. When uneven FX swaps are traded, the domestic interest rate has to be agreed upon as well.

After this short analysis, we are able to sum up the specific features of outright and FX swap contracts:

1. An outright contract is exposed to an FX rate risk for the full nominal amount. It also has exposure to interest rates, although this is very small compared to the FX risk.

2. In an FX swap contract the FX rate risk of the spot transaction is almost entirely offset by the outright transaction. In the case of non-par contracts, the FX risk is completely offset, and only a residual exposure to the interest rate risk is left.
3. For the reasons above, outright contracts are mainly traded by speculators and hedgers in the FX market.
4. The FX swap is rather a treasury product, traded in the interbank market to move funds from one currency to another, without any FX risk (for par contracts), and to hedge or get exposure to the interest rate risks in two different currencies. Nonetheless, it is used by options traders to hedge exposure to the domestic and foreign interest rates.

Remark 1.2.2. *If we assume that we are working in a world where the occurrence of default of a counterparty is removed, then by standard arbitrage arguments we must impose that the forward points of an outright contract are exactly the same as the swap points of an FX swap contract. Things change if we introduce the chance that market operators can go bankrupt, so that the mechanics of the two contracts imply great differences in their pricing.*

We have seen before that the arbitrage argument of the replica Strategy 1.2.1 can no longer be applied when default is taken into account, so that the actual traded forward price can differ substantially from the theoretical arbitrage price, since a trader can suffer a big loss if the counterparty from whom they bought the deposit defaults. Now, we would like to examine whether removing the no-default assumption impacts in the same way both the outright and the FX swap contract.

To this end, consider the case when the FX swap points for a given expiry imply a tradable forward price $F'(t, T)$ greater than the theoretical price $F(t, T)$ obtained by formula (1.1). *To exploit the possible arbitrage, we could borrow one million units of foreign currency, say the euro, and close an FX swap contract "sell and buy back 1 mio EUR, uneven amount", similar to that in Example 1.2.2, but with a reverse sign. Basically, we are operating Strategy 1.2.1 with an FX swap, instead of an outright contract. Assume also that, after the deal is struck, our counterparty in the FX swap deal might be subject to default, in which case they will not perform their contractual obligations, so we will not receive back the one million euros times $(1 + r^f \tau)$, against $F'(t, T)$ million US dollars times $(1 + r^f \tau)$ paid by us. In such an event, we will not have the amount of money we need to pay back our loan in euros, whose value at the end of the contract is equal to $(1 + r^f \tau)$ million euros. Nevertheless, we still have the initial exchanged amount in USD, equal to S_t (the FX spot rate at inception of the contract), and we could use this to pay back our debt. In this case, assuming we have kept the amount in cash, we can convert it back into euros at the terminal FX spot rate S_T, which might be lower or higher than S_t, so that we can end up with a final amount of euros greater or smaller than one million (the euro amount will be S_t/S_T). The terminal economic result could be a profit or a loss, depending on the level of the FX spot rate S_T and on how much we have to pay for the interest on the loan in euros. Nonetheless, we may reasonably expect not to lose as much as one million euros, and the total loss (or even profit) is a function of the volatility of the exchange rate and the time to maturity of the contract.*

Assume now that we operated Strategy 1.2.1 with an outright contract. We borrow one million euros, convert it into dollars at S_t, buy a deposit in dollars, and convert the terminal amount by selling an outright at the rate $F'(t, T)$. If our counterparty defaults, they will not pay back the amount of money we lent to them (supposing there is no fraction of the notional amount recovered) and we will end up with no money to sell via the outright, so as to convert it into euros and pay back our loan. In this case we are fully exposed to the original amount *of one million euros and we will suffer a loss for sure equal to this amount, plus the interest on the loan.*

From the two cases we have described, we can see that the FX swap can be considered as a collateralized loan. The example shows a situation just as if we lent an amount denominated in euros, collateralized by an amount denominated in dollars. Clearly, the collateral is not risk-free, since its value in euros is dependent on the level of the exchange rate, but it is a guarantee that will grant a presumably high recovery rate of the amount lent on the occurrence of default of the counterparty, and we could possibly end up with a profit. In the other case we examined, that is the outright contract, we see that we have no collateral at all as a guarantee against the default of the counterparty, so we are fully exposed to the risk of losing the amount of dollars we lent to them. This loss can be mitigated if we assume that we can recover a fraction of the notional amount we lent, but the recovery will very likely be much smaller than the fraction of notional we can recover via the collateral.

There are two conclusions we can draw:

1. *The forward rate $F(t, T)$ determined as in equation* (1.1) *does not identify the unique arbitrage-free price of an outright contract, if we include the chance of default of the counterparty.*
2. *The forward price implied by an FX swap contract can be different from that of an outright contract when default of the counterparty is considered, because Strategy 1.2.1 operated with an FX swap is less risky than the same strategy operated with an outright contract.*

1.3 FX OPTION CONTRACTS

FX options are no different from the usual options written on any other asset, apart from some slight distinctions in the jargon. The definition of a *plain vanilla European* option contract is the following:

Definition 1.3.1. ***European plain vanilla FX option contract.*** *Assume we have the pair XXXYYY. Two counterparties entering into a plain vanilla FX option contract agree on the following, according to the type of option traded:*

- *Type* ***XXX call YYY put****: the buyer has the right to enter at expiry into a spot contract to buy (sell) the notional amount of the XXX (YYY) currency, at the strike FX rate level K.*
- *Type* ***XXX put YYY call****: the buyer has the right to enter at expiry into a spot contract to sell (buy) the notional amount of the XXX (YYY) currency, at the strike FX rate level K.*

The spot contract at expiry is settled on the settlement date determined according to the rules for spot transactions. The notional amount N in the XXX base currency is exchanged against $N \times K$ units of the numeraire currency. The buyer pays a premium at inception of the contract for their right.

The following chapters are devoted to the fair calculation of the premium of an option, the analysis of the risk exposures engendered by trading it, and the possible approaches to hedging these exposures. Clearly, this will be done not only for plain vanilla options, but also for other kinds of options, usually denoted as *exotics*. A very rough taxonomy for FX options is presented in Table 1.2; this should be considered just as a guide to how the analysis will be organized in what follows. Besides, it is worth noticing that the difference between

Table 1.2 Taxonomy of FX options

Group	Name	Exercise	Monitoring
Plain vanilla	Call/put	E/A	E
First-generation exotic	Digital	E	E
First-generation exotic	Knock-in/out barriers	E/A	E/C/D
First-generation exotic	Double-knock in/out barriers	E/A	E/C/D
First-generation exotic	One-touch/no-touch/ double-no-touch/double-touch	A	C/D
First-generation exotic	Asian	E/A	D
First-generation exotic	Basket	E/A	D
Second-generation exotic	Window knock-in/out barriers	E/A	E/C/D
Second-generation exotic	First-in-then-out barriers	E/A	E/C/D
Second-generation exotic	Forward start plain/barriers	E/A	E/C/D
Second-generation exotic	External barriers	E/A	E/C/D
Second-generation exotic	Quanto plain/barriers	E/A	E/C/D

Exercise: European (E), American (A). Monitoring: at expiry (E), continuous (C), discrete (D).

first-generation and second-generation exotics is due to the time sequence of their appearance in the market rather than any reference to their complexity.

It is worth describing in more detail the option contract and the market conventions and practices relating to it.

1.3.1 Exercise

The exercise normally has to be announced by the option's buyer at 10:00 AM New York time; options are denominated *NY Cut* in this case, and they are the standard options traded in the interbank market. The counterparties may also agree on a different time; such as 3:00 PM Tokyo time; in this case we have the *Tokyo Cut*. The exercise is considered automatic for a given percentage of in-the-moneyness of the options at expiry (e.g., 1.5%), according to the ISDA master agreement signed between two professional counterparties before starting any trading activity between them. In other cases the exercise has to be announced explicitly, although it is market fairness to consider exercised (or abandoned) options manifestly in-the-money (or out-of-the money), even without any call from the option's buyer.

1.3.2 Expiry date and settlement date

The expiry date for an option can be any date when at least one marketplace is open, then the settlement date is set according to the settlement rules used for spot contacts. Some market technicalities concern the determination of the expiry and settlement (delivery) dates for what we call *canonic* or *standard* dates. In more detail, in the interbank market daily quotes are easily available for standard expiries expressed in terms of time units from the trade date, i.e., overnight, weeks, months and years.

Day periods. Overnight is the simplest case to analyse, since it indicates an expiry for the next available business day, so:

1. In normal conditions it is the day after the trade date or after three days in case the trade date is a Friday (due to the weekend).

2. The expiry is shifted forward if the day after the trade date is not a business day all around the world (e.g., 25 December). On the contrary if at least one marketplace is open, then the expiry date is a good one.
3. Once the expiry date is determined, the settlement date is calculated with the rules applied for the spot contract.

If the standard expiry is in terms of number of days (e.g., three days), the same procedure as for overnight applies, with expiry date initially and tentatively set as the number of days specified after the trade date.

Week periods. This case is not very different from the day period one:

1. The expiry is set on the same week day (e.g., Tuesday) as the trade date, for the given number of weeks ahead in the future (e.g., 2 for two weeks).
2. At least one marketplace must be open, otherwise the expiry is shifted forward by one day and the open market condition checked again.
3. Once the expiry is determined, the usual rules for the spot contract settlement date apply.

Month and year periods. In these cases a slightly different rule applies, since the spot settlement date corresponding to the trade date is the driver. More specifically:

1. One moves ahead in the future by the given number of periods (e.g., 6 for six months), then the same day of the month as the spot settlement date (corresponding to the trade date, in the current month) is taken as the settlement date of the option (e.g., again for six-month expiry, if the trade date is the 13th of the current month and the 15th is the settlement date for a corresponding spot contract, then the 15th day of the sixth month in the future will be the option settlement date). If the settlement date of the future month is not a valid date for the pair involved, then the date is shifted forward until a good date is achieved.
2. If the settlement determined in (1) happens to fall in the month after the one corresponding to the number of periods considered (e.g., the six-month expiry yields a settlement actually falling in the seventh month ahead), then the *end-of-month* rule applies. From the first settlement date (identified from the spot settlement of the trade date), the date is shifted backward until a valid (for the contract's pair) settlement date is reached.
3. The expiry can now be calculated by applying backward from the settlement date the rules for a spot contract.
4. The year period is treated with same rules simply by considering the fact that one year equals 12 months.

We provide an example to clarify the rules listed above.

Example 1.3.1. *Assume we trade an option EUR call USD put with expiry in one month. We consider the following cases:*

- *The trade date is* 19 *October 2007. From the market calendars the spot settlement date for such a trade date can be calculated and set on* 23 *October so that the settlement of the option has to be set on* 23 *November (i.e., the same day one month ahead). This date can be a settlement date for the EURUSD pair and the corresponding expiry date is* 21 *November, since the* 22*nd is a holiday in the USA but is counted as a business day according to the spot date rules. Actually, we know from Example 1.1.2 that the spot trades dealt on* 20 *November also imply a settlement date on the* 22*nd. When the expiry date is calculated* *working backward from the settlement, the first possible trade date encountered is taken (i.e., the* 21*st in this case).*

- *The trade date is* 19 *October 2007. From the market calendars the spot settlement date for such a trade date is* 24 *October, thus the option's settlement date is* 24 *November, which is a Saturday, so it is shifted forward to the first available business day for both currencies: Monday* 26 *November. Working backward to calculate the expiry date, we would take* 22 *November but this is a US holiday, so we move one more day backward and set the expiry on the* 21*st, which agrees with spot settlement rules.*

After analysing the rules for standard expiries, for the sake of completeness we just remark that if a specific date is agreed upon for the expiry (e.g., 7 January 2008), then the standard spot settlement rules apply to calculate the option's settlement date (9 January, if the contract's pair is EURUSD).

1.3.3 Premium

The option's premium is paid on the spot settlement date corresponding to the trade date. It can be paid in one of either currencies of the underlying pair and it can be expressed in four different ways, which we list below:

1. *Numeraire currency units* (p_{numccy}). This is the standard way in which, for some pairs, premiums are expressed for plain vanilla options in the interbank market after the closing of the deal. It is worth noticing also that this is the natural premium one calculates by a pricing formula. The actual premium to pay is calculated by multiplying the currency units times the notional amount (in base currency units): $N \times p_{numccy}$.
2. *Numeraire currency percentage* ($p_{numeccy\%}$). This is the standard way in which premiums are expressed and quoted for exotic (one-touch, double-no-touch, etc.) options in the interbank market, when the payout is a numeraire currency amount. It can be calculated by dividing the premium in numeraire currency units by the strike: $p_{numccy\%} = \frac{p_{numccy}}{K} \times 100$. The actual premium to pay is equal to the notional amount in numeraire currency units ($N \times K$) times the numeraire currency percentage premium: $N_{numccy} \times \frac{p_{numccy\%}}{100}$.
3. *Base currency units* ($p_{baseccy}$). This way of quoting may be useful when the numeraire currency amount is fixed for all the options entering into a given strategy (e.g., in an EUR call USD put spread). It can be calculated by dividing the premium in numeraire currency units by the spot FX rate and then by the strike: $p_{baseccy} = \frac{p_{numccy}}{S_t K}$. The actual premium to pay is equal to the notional amount, expressed in numeraire currency (that is: $N \times K$), times the base currency units premium: $N_{numccy} \times p_{baseccy}$.
4. *Base currency percentage* ($p_{baseccy\%}$). This is the standard way in which premiums are expressed and quoted for exotic (barrier) options, and for some pairs also for plain vanilla options, in the interbank market. It can be calculated by dividing the premium in numeraire currency units by the spot FX rate: $p_{baseccy\%} = \frac{p_{numccy}}{S_t} \times 100$. The actual premium to pay is equal to the notional amount times the base currency percentage premium: $N \times \frac{p_{baseccy\%}}{100}$.

In Table 1.3 we report some market conventions for option premiums; usually, the numeraire currency premium is multiplied by a factor such that it is expressed in terms of pips (see above for the definition of the latter), or as a percentage of either notional rounded to the nearest quarter of 0.01%. We will see later that the way markets quote premiums has an impact on the building of the volatility matrix, so that it is not just a curiosity one may lightly neglect.

Table 1.3 Market conventions for option premiums for some pairs

Pair	p_{numccy}	$p_{baseccy}\%$
EURUSD	USD pips	
EURCAD	CAD pips	
EURCHF		EUR %
EURGBP	GBP pips	
EURJPY		EUR %
EURZAR		EUR %
GBPCHF		GBP %
GBPJPY		GBP %
GBPUSD	USD pips	
USDCAD		USD %
USDCHF		USD %
USDJPY		USD %
USDZAR		USD %

Example 1.3.2. *Assume we want to buy* 2 000 000 *EUR call USD put struck at* 1.3500*, with a reference EURUSD spot rate equal to* 1.2800*. The notional amount in USD is* $2\,000\,000 \times 1.3500 = 2\,700\,000$*. The premium can be quoted in one of the four ways we have examined and we have that:*

1. *If the premium is in numeraire currency units and it is* $p_{USD} = 0.0075$ *US dollars per one EUR unit of option, we will pay* $2\,000\,000 \times 0.0075 = 15\,000$ *USD.*
2. *If the quotation is expressed as a numeraire currency percentage, the premium is* $p_{USD\%} = \frac{0.0075}{1.3500} \times 100 = 0.5550\%$ *(rounded to the nearest quarter of* 0.01%*) for one USD unit of option dollar, and we pay* $0.5550 \times \frac{2\,700\,000}{100} = 14\,985$ *USD (the small difference of* 15 000 *is due to rounding conventions).*
3. *If the quotation is in base currency units, the premium is* $p_{EUR} = \frac{0.75}{1.2800 \times 1.3500} = 0.00435$ *EUR per one USD unit of option dollar, and we pay* $0.55 \times \frac{2\,700\,000}{100} = 11\,750$ *EUR.*
4. *Finally, if the premium is expressed as a base currency percentage, it is* $p_{EUR\%} = \frac{0.0075}{1.2800} \times 100 = 0.5875\%$ *of the EUR notional (rounded to the nearest quarter of* 0.01%*) and we pay* $0.5875 \frac{2\,000\,000}{100} = 11\,750$ *EUR.*

1.3.4 Market standard practices for quoting options

FX options can be dealt for any expiry and also for any level of strike price. Amongst professionals, options are quoted according to standards: some of them are actually rather clever, and make FX options one of the most efficient OTC derivatives markets.

Let us start with plain vanilla options. Firstly, options are usually quoted for standard dates, although it is possible to ask a market maker for an expiry occurring on any possible date. Secondly, quotations are not in terms of (any of the four above) premiums but in terms of implied volatilities, that is to say, in terms of the volatility parameter to plug into the BS model (given the values of all the other parameters and the level of the FX spot rate, retrievable from the market). Once the deal is closed, the counterparties may agree to actually express the premium in any of the four ways listed above, although the standard way is in numeraire