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Exotic Options and Hybrids

A Guide to Structuring, Pricing and Trading

MOHAMED BOUZOUBAA
ADEL OSSEIRAN

Further praise for Exotic Options and Hybrids

"This book brings a practitioner's prospective into an area that has seen little treatment to date. The challenge of writing a logical, rigorous, accessible and readable account of a vast and diverse field that is structuring of exotic options and hybrids is enthusiastically taken up by the authors, and they succeed brilliantly in covering an impressive range of products."

Vladimir Piterbarg, Head of Quantitative Research, Barclays

"What is interesting about this excellent work is that the reader can measure clearly that the authors are sharing a concrete experience. Their writing approach and style bring a clear added value to those who want to understand the structuring practices, Exotics pricing as well as the theory behind these."

Younes Guemouri, Chief Operating Officer, Sophis

"The book provides an excellent and compressive review of exotic options. The purpose of using these derivatives is well exposed, and by opposition to many derivatives' books, the authors focus on practical applications. It is recommended to every practitioner as well as advanced students looking forward to work in the field of derivatives."

Dr Amine Jalal, Vice President, Equity Derivatives Trading, Goldman Sachs International

"*Exotic Options and Hybrids* is an exceptionally well written book, distilling essential ingredients of a successful structured products business. Adel and Mohamed have summarized an excellent guide to developing intuition for a trader and structurer in the world of exotic equity derivatives."

Anand Batepati, Structured Products Development Manager, HSBC, Hong Kong

"A very precise, up-to-date and intuitive handbook for every derivatives user in the market."

Amine Chkili, Equity Derivatives Trader, HSBC Bank PLC, London

"*Exotic Options and Hybrids* is an excellent book for anyone interested in structured products. It can be read cover to cover or used as a reference. It is a comprehensive guide and would be useful to both beginners and experts. I have read a number of books on the subject and would definitely rate this in the top three."

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Idriss Amor, Rates and FX Structuring, Bank of America, London

"*Exotic Options and Hybrids* is an accessible and thorough introduction to derivatives pricing, covering all essential topics. The reader of the book will certainly appreciate the alternation between technical explanations and real world examples."

Khaled Ben-Said, Quantitative Analyst, JP Morgan Chase, London

"A great reference handbook with comprehensive coverage on derivatives, explaining both theory and applications involved in day-to-day practices. The authors' limpid style of writing makes it a must-read for beginners as well as existing practitioners involved in day-to-day structuring, pricing and trading."

Anouar Cedrati, Structured Products Sales, HSBC, Dubai

"A good reference and an excellent guide to both academics and experts for its comprehensive coverage on derivatives through real world illustrations and theory concepts."

Abdessamad Issami, Director of Market Activities, CDG Capital

"*Exotic Options and Hybrids* offers a hands-on approach to the world of options, giving good insight into both the theoretical and practical side of the business. A good reference for both academics and market professionals as it highlights the relationship between theory and practice."

Joseph Nehme, Bachelor of Engineering AUB, ESSEC MBA,
Equity Derivatives Marketer, Merrill Lynch, London

"A great guide for experienced professionals or those just starting out in the space. Both the core concepts of structured derivatives as well as the more complex exotic's pricing and management come across with great lucidity. *Exotic Options and Hybrids* is a great complement to anybody's financial library."

Nabil Achtioui, Volatility Arbitrage Trader, Calyon, Paris

"*Exotic Options and Hybrids* serves as a good introduction into the world of structured equities and hybrids, and would be useful for both the enthusiastic novice as well as the seasoned professional who wants to recall a few concepts. Highly recommended."

Rahul Karkun, Rates and Hybrid Structuring, Bank of America, London

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A Guide to Structuring, Pricing and Trading

Mohamed Bouzoubaa and Adel Osseiran

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To my parents
Chakib and Fadia
MB

To the memory of my grandfather
Adil
AO

Contents

List of Symbols and Abbreviations

$\mathbf{1}$	Indicator function
ATM	At the money
ATMF	At the money forward
bp	Basis point, equal to 1% of 1%
EUR	Euro
GBP	Great Britain pound
ITM	In the money
JPY	Japanese yen
K	The strike of a specified option
MTM	Marked-to-market
$\mathcal{N}$	Normal cumulative distribution function
OTC	Over the counter
OTM	Out of the money
ρ	Correlation
q	Dividend yield of a specified asset
r	Risk-free rate of interest
$S(t)$	Price of asset S at time t
$S_i(t)$	Price of asset S_i at time t (multi-asset case)
σ	The volatility of a specified asset
T	Maturity of an option
USD	United States dollar

Preface

Toxic waste. . .it is a sad day when derivatives are described as toxic waste. Are these financial products really so, particularly those of exotic nature, or is it in fact people's grasp and usage of them that is the source of toxicity? While the use of derivatives increased in recent years at astounding rates, the crash of 2008 has revealed that people's understanding of them has not rivalled their spread. *Exotic Options and Hybrids* covers a broad range of derivative structures and focuses on the three main parts of a derivative's life: the structuring of a product, its pricing and its hedging. By discussing these aspects in a practical, non-mathematical and highly intuitive setting, this book blasts the misunderstandings and the stigma, and stands strong as the only book in its class to make these *exotic* and complex concepts truly accessible.

We base *Exotic Options and Hybrids* on a realistic setting from the heart of the business: inside a derivatives operation. Working from the assumption that one has a range of correctly implemented models, and the ability to trade a set of basic financial instruments, a client's need for a tailored financial product then raises these questions: How does one structure this product, correctly price it for the sale, and then hedge the resulting position until its maturity? Following a risk-centred approach, *Exotic Options and Hybrids* is a well-written, thoroughly researched and consistently organized book that addresses these points in a down-to-earth manner.

The book contains many examples involving time series and scenarios for different assets, and while hypothetical, all are carefully designed so as to highlight interesting and significant aspects of the business. Adoptions of real trades are examined in detail. To further illuminate payoff structures, their introduction is accompanied by payoff diagrams, scenario analyses involving figures and tables of paths, plus lifelike sample term sheets. By first understanding the investor's point of view, readers learn the methodology to structure a new payoff or modify an existing one to give different exposures. The names of various products can sometimes vary from one side of the industry to another, but those attributed to the products discussed in this book are commonly accepted to a great extent. Next, the reader learns how to spot where the risks lie to pave the way for sound valuation and hedging of the products. Models are de-mystified in separately dedicated sections, but their implications are alluded to throughout the book in an intuitive and non-mathematical manner.

Exotic Options and Hybrids is the first book to offer insights into the structuring, pricing and trading of modern exotic and hybrid derivatives, without complicating matters with the use of maths. The applications, the strengths and the limitations of various models are highlighted, in relevance to the products and their risks, rather than the model implementations. Readers can

thus understand how models work when applied to pricing and hedging, without getting lost in the mathematical dwellings that shape related texts. While previous texts are heavily technical, others do not offer enough exposure, if any, to the more advanced and modern structures. The multitude of structures covered in *Exotic Options and Hybrids* is quite comprehensive, and encompasses many of the most up-to-date and promising products, including hybrid derivatives and dynamic strategies.

The book is formed of four parts, each containing related chapters which evolve in increasing degrees of complexity in the structures. Readers will be continuously stimulated by more advanced topics, and because of this breakdown the book can be read from front to back without loss of interest. Alternatively, readers can jump straight to a specific chapter because the book is self-contained and references to earlier chapters and sections within the book are explicitly clear. Furthermore, movement between the various angles of analysis of a specific product or concept is transparent, leaving readers free to focus on one aspect, or to read an entire treatment of a subject.

The first two chapters lay the foundations and explain not only the basic blocks of derivatives but also the setup and people involved in the creation, pricing and hedging of exotic structures. Chapters 3 to 7 define vanilla options, the risks involved in trading them and the different tools one can use to measure them. The second part of the book deals with the concept of dispersion which is of key importance in the world of exotic options. Chapters 10 and 11 focus on barrier options and digitals that are very much used in the conception of structured products. Chapters 13 to 16 constitute the third part of the book and present cliquets and related structures, mountain range options, and volatility derivatives, all of which are considered to be slightly more advanced exotic products.

After completing the discussion of exotic structures based upon equities, we move to hybrid derivatives. These chapters allow us to draw on many of the points made earlier in the book regarding correlation, dispersion and volatility, and provide a transparent insight into the world of hybrid derivatives. The first two of the four chapters on hybrids discuss the key asset classes: interest rates, commodities, foreign exchange, inflation and credit. For each asset class we look at the markets individually and gain insight into the nature of each, the various underlyings, vanilla instruments, skews and smiles and a brief look at some popular exotics in each. These are followed by a chapter that discusses the structuring of hybrid derivatives and explains how to construct meaningful combinations of the various asset classes. The last chapter on hybrids discusses the pricing intricacies of these instruments, starting from each asset class and then modelling combinations thereof. Chapter 21, the final chapter, deals with thematic indices and dynamic strategies. These assets are very different from the traditional structured products presented throughout the book, and constitute the new generation of advanced investment solutions.

We strongly believe that attentive readers of this book will learn many valuable insights in to all facets of the business of structured products. *Exotic Options and Hybrids* appeals to all the parties involved in the creation, pricing and hedging of the simplest to the most complex products. Once the heart of the business and its technical features are deeply assimilated, readers should be well equipped to contribute their own stone to the world of structured products.

Part I
Foundations

1

Basic Instruments

Concentrate all your thoughts on the task at hand. The sun's rays do not burn until brought to a focus.

Alexander Graham Bell

1.1 INTRODUCTION

We begin the book by first reviewing the basic set of financial instruments. These are either building blocks of derivatives or impact their valuation. A derivative is a financial instrument *derived* from another asset. It can also be derived from a set of events, an index or some condition, and in all cases we refer to these as the underlying asset(s) of the derivative. The set of financial instruments discussed in this introductory chapter fall into two categories: they are either *exchange traded* or *over the counter*. Exchange-traded products, also referred to as *listed*, are standardized products that are traded on an exchange which acts as the intermediary. Futures contracts are an example of exchange-traded contracts. Over-the-counter products, on the other hand, are privately agreed directly between two parties, without the involvement of an exchange. This includes almost all swaps and exotic derivatives.

We first look at interest rates and explain the differences between the various types. These include LIBOR, which is not only the most common floating rate used in swap agreements but also a reference rate that can be used to compute the present value of a future amount of money. We also introduce the different discounting methods, which are of prime importance in the valuation of derivatives. Within the topic of fixed income, we define the essential debt instruments known as zero coupon bonds.

This chapter also provides the basics of equity and currency markets. The features of stocks are defined as well as the parameters impacting their future price. We discuss how a currency can be viewed as a stock asset; we then define the importance and uses of indices and exchange-traded funds in trading strategies. Forward and futures contracts are also described in this chapter.

To round out the review of financial instruments we discuss swaps, which are agreements that occupy a central and crucial position in the over-the-counter market; the most commonly traded swap being the interest rate swap. After defining swaps' features and trading purposes, we introduce cross-currency swaps that are used to transform a loan from one currency to another. Finally, we present the features of total return swaps, which can replicate the performances of assets such as equities or bonds.

1.2 INTEREST RATES

Interest rates represent the premium that has to be paid by a borrower to a lender. This amount of money depends on the credit risk – that is, the risk of loss due to a debtor's non-payment of his duty, on the interest and/or the principal, to the lender as promised. Therefore, the higher

the credit risk, the higher the interest rates charged by the lender as compensation for bearing this risk.

Interest rates play a key role in the valuation of all kinds of financial instruments, specifically, interest rates are involved to a large extent in the pricing of all derivatives. For any given currency, there are many types of rates that are quoted and traded. Therefore, it is important to understand the differences between these rates and the implications of each on the valuation of financial instruments.

1.2.1 LIBOR vs Treasury Rates

Among the more popular rates, we find Treasury rates and LIBOR rates. Treasury rates are the rates earned from bills or bonds issued by governments. Depending on the issuing sovereign body, these can be considered as risk-free rates since it is assumed that certain governments will not default on their obligations. However, derivatives traders may use LIBOR rates as short-term risk-free rates instead of Treasury rates.

The London Interbank Offered Rate (LIBOR) is the interest rate at which a bank offers to lend funds to other banks in the interbank market. LIBOR rates can have different maturities corresponding to the length of deposits and are associated with all major currencies. For instance, 3-month EURIBOR is the rate at which 3-month deposits in euros are offered; 12-month US LIBOR is the rate at which 12-month deposits in US dollars are offered; and so on. LIBOR will be slightly higher than the London Interbank Bid Rate (LIBID), which is the rate at which banks will accept deposits from other financial institutions.

Typically, a bank must have an AA credit rating (the best credit rating given by the rating agency Standard and Poor's being AAA) to be able to accept deposits at the LIBOR rate. A rating as such would imply that there is a small probability that the bank defaults. This is why LIBOR rates are considered to be risk free although they are not totally free of credit risk. Moreover, a number of regulatory issues can impact the value of Treasury rates and cause them to be consistently low. For this reason, LIBOR is considered by derivatives traders to be a better measurement of short-term risk-free rates than Treasury rates. In the world of derivatives, people think directly of LIBOR rates when talking about risk-free rates.

The difference between the interest rate of 3-month Treasury bills and the 3-month LIBOR is known as the TED spread, and can be used as a measure of liquidity in interbank lending. LIBOR, which corresponds to interbank lending, compared to the risk-free rates of Treasury bills is an indication of how willing banks are to lend money to each other. LIBOR rates involve credit risk, whereas Treasury rates do not, and thus the TED spread serves as a measure of credit risk in the interbank market. Higher TED spreads correspond to higher perceived risks in lending, and vice versa.

1.2.2 Yield Curves

For any major currency, the interest rates paid on bonds, swaps or futures are closely watched by traders and plotted on a graph against their maturities. These graphs are commonly called yield curves and they emphasize the relationship between interest rates and maturity for a specific debt in a given currency. The points on the curve are only known with certainty for specific maturity dates; the rest of the curve is built by interpolating these points.

For each currency, there are several types of yield curves describing the cost of money depending on the creditworthiness of debtors. The yield curves showing interest rates earned by the holders of bonds issued by governments are called government bond yield curves. Besides these curves, there are corporate curves that correspond to the yields of bonds issued by companies. Because of a higher credit risk, the yields plotted in corporate curves are usually higher and are often quoted in terms of a credit spread over the relevant LIBOR curve. For instance, the 10-year yield curve point for Renault might be quoted as LIBOR + 75 bp (a basis point or bp being equal to 0.01%), where 75 bp is the credit spread. In order to price a financial instrument, a trader will choose the yield curve that corresponds to the type of debt associated with this instrument. Despite there being different time-periods corresponding to the various rates, they are typically expressed as an annual rate. This allows interest rates to be compared easily.

Yield curves are typically upwards sloping, with longer term rates higher than shorter term rates. However, under different market scenarios the yield curve can take several different shapes, being humped or possibly downward sloping. We go into much further detail regarding the shapes of yield curves when we discuss interest rates in the context of hybrid derivatives in Chapter 17. Credit spreads are also discussed in more detail in Chapter 18 in the context of defaultable bonds and credit derivatives.

1.2.3 Time Value of Money

The concept of the *time value* of money is key to all of finance, and is directly related to interest rates. Simply put, an investor would rather take possession of an amount of money today, for example $1,000, than take hold of the $1,000 in a year, 10 years, or even one week. In fact, the concept of interest over an infinitesimally small period arises, and the preference is that an investor would rather have the money now than at any point in the future. The reason is that interest can be earned on this money, and receiving the exact same amount of money at a time in the future is a forfeited gain.

One hundred dollars to be paid one year from now (a future value), at an expected rate of return of $i = 5\%$ per year, for example, is worth in today's money, i.e. the *present value*:

$$\mathrm{PV} = \mathrm{FV} \times \frac{1}{(1+i)^n} = \frac{100}{1.05} = 95.24$$

So the present value of 100 dollars one year from now at 5% is $95.24. In the above equation $n = 1$ is the number of periods over which we are compounding the interest. An important note is that the rate i is the interest rate for the relevant period. In this example we have an annual rate applied over a 1-year period. Compounding can be thought of as applying the interest rate to one period and reinvesting the result for another period, and so on.

To correctly use interest rates we must convert a rate to apply to the period over which we want to compute the present value of money. Interest rates can be converted to an equivalent continuous compounded interest rate because it is computationally easier to use. We can think of this as compounding interest over an infinitesimally small period. The present value, PV, at time 0 of a payment at time t in the future, is given in terms of the future value, FV, and the continuously compounded interest rate r by

$$\mathrm{PV} = \mathrm{FV}\mathrm{e}^{-rt}$$

Exercise

Consider you make a deposit of \$100 today. Let's assume that interest rates are constant and equal to 10%. In the case of annual compounding, how many years are needed for the value of the deposit to double to \$200?

Discussion

Let y denote the number of years needed to double the initial investment. Then: $\text{FV} = \text{PV} \times (1+i)^y$. The present value formula can be rearranged such that

$$y = \frac{\ln(\text{FV}/\text{PV})}{\ln(1+i)} = \frac{\ln(200/100)}{\ln(1.10)} = \frac{0.693}{0.0953} = 7.27$$

years[1].

This same method can be used to determine the length of time needed to increase a deposit to any particular sum, as long as the interest rate is known.

1.2.4 Bonds

A bond is a debt security used by governments and companies to raise capital. In exchange for lending funds, the holder of the bond (the buyer) is entitled to receive coupons paid periodically as well as the return of the initial investment (the principal) at the maturity date of the bond. The coupons represent the interest rate that the issuer pays to the bondholders in exchange for holding their debt. Usually, this rate is constant throughout the life of the bond; this is the case of fixed rate bonds. The coupons can also be linked to an index; we then talk about floating rate notes. Common indices include money market indices, such as LIBOR or EURIBOR, or CPI (the Consumer Price Index) inflation rate linked bonds. Bonds can have a range of maturities classified as: short (less than 1 year), medium (1 to 10 years) and long term (greater than 10 years). In this section we now focus on fixed rate bonds.

The market price of a bond is then equal to the sum of the present values of the expected cashflows. Let t denote the valuation date and C_i the value of the coupons that are still to be paid at coupon dates t_i, where $t \leq t_i \leq t_n = T$. The value of a bond is then given by the following formula:

$$\text{Bond}(t,T) = \sum_{i=1}^{n} C_i B(t,t_i)$$

which results in

$$\text{Bond}(t,T) = \sum_{i=1}^{n} C_i \mathrm{e}^{-r(t,t_i)\times(t_i-t)}$$

The price of a bond can be quoted in terms of a normal price as shown above or in terms of yield to maturity y, which represents the current market rate for bonds with similar features.

[1] This is often referred to as *The Rule of 72*.

Yield to maturity is defined as follows:

$$\text{Bond}(t, T) = \sum_{i=1}^{n} C_i \mathrm{e}^{-y \times (t_i - t)}$$

The market price of a bond may include the interest that has accrued since the last coupon date. The price, including accrued interest, is known as the dirty price and corresponds to the fair value of a bond, as shown in the above formula. It is important to note that the dirty price is the price effectively paid for the bond. However, many bond markets add accrued interest on explicitly after trading. Quoted bonds, such as those whose prices appear in the *Financial Times* are the clean prices of these bonds.

$$\text{Clean Price} = \text{Dirty Price} - \text{Accrued Interest}$$

Bonds are commonly issued in the primary market through underwriting. Once issued, they can then be traded in the secondary market. Bonds are generally considered to be a safer investment than stocks due to many reasons, one being that bonds are senior to stocks in the capital structure of corporations, and in the event of default bondholders receive money first. Bonds can pay a higher interest compared to stocks' dividends. Also, bonds generally suffer from less liquidity issues than stocks. In times of high volatility in the stock market, the bond can serve as a diversification instrument to lower volatility.

Nonetheless, bonds are not free of risk, because bond prices are a direct function of interest rates. In fact, fixed rate bonds are attractive as long as the coupons paid are high compared to the market rates, which vary during the life of the product. Consequently, bonds are subject to interest rate risk, since a rise in the market's interest rates decreases the value of bonds and vice versa. We can also understand this effect by looking at the bond price formula: if the interest rate used to discount the coupons goes up, their present value goes down and the price of the bond decreases. Alternatively, if interest rates go down, bond prices increase.

Moreover, bond prices depend on the credit rating of the issuer. If credit rating agencies decide to downgrade the credit rating of an issuer, this causes the relevant bonds to be considered a riskier investment, therefore a bondholder would require a higher interest for bearing greater credit risk. Since the coupons are constant, the price of the bond decreases. Therefore, credit risk increases the volatility of bond prices. When turning to some government bonds (for example, US Treasuries), one considers these to be risk free, but any deviation from these in terms of creditworthiness will be reflected in the price as an added risk.

In the case of callable bonds, the bond can be *called*, i.e. bought back, by the issuer at a pre-specified price during some fixed periods laid out in the contract. The bondholder is subject to reinvestment risk. Buying a callable bond is equivalent to buying a bond and selling an American call option on this bond. When interest rates go down, the bond's price goes up and the issuer is more likely to exercise his call option and buy back his bond. The bondholder would then have to reinvest the money received earlier; but in such a scenario, with lower interest rates, it would be hard to enter into a better deal.

1.2.5 Zero Coupon Bonds

Zero coupon bonds are debt instruments where the lender receives back a principal amount (also called face value, notional or par value) plus interest, only at maturity. No coupons are paid during the life of the product, thus the name. In fact the interest is deducted up front and

is reflected in the price of the zero coupon bond since it is sold at a discount, which means that its price is lower than 100% of the notional. Issuing zero coupon bonds is advantageous from a medium-term liquidity perspective, compared to issuing coupon-bearing bonds in which payments will have to be made at various points in the life of the bond. A US Treasury Bill is an example of a zero coupon bond.

The price of a zero coupon is equal to the present value of the par value, which is the only cashflow of this instrument and paid at maturity T. Zero coupon bonds are tradeable securities that can be exchanged in the secondary market. Let $B(t, T)$ denote the price in percentage of notional of a zero coupon bond at time t. Depending on the discounting method used by a trader to compute the interest amount, $B(t, T)$ is directly related to interest rates by the following formulas:

Linear: Interest is proportional to the length of the loan

$$B(t, T) = \frac{1}{1 + r(t, T) \times (T - t)}$$

Actuarial: Interest is compounded periodically

$$B(t, T) = \frac{1}{(1 + r(t, T))^{T-t}}$$

Continuous: Interest is compounded continuously

$$B(t, T) = \mathrm{e}^{-r(t,T)\times(T-t)}$$

Here $r(t, T)$ stands for the appropriate interest rate at time t and maturity $(T - t)$, which is the time to maturity of the loan expressed in years.

Also note that in order to compute, at time t, the present value of any cashflow that occurs at time T, one must multiply it by $B(t, T)$. From now on, we are going to use continuous compounding to discount cashflows for the valuation of derivatives.

1.3 EQUITIES AND CURRENCIES

1.3.1 Stocks

Companies need cash to operate or finance new projects. It is often the case that their cash income does not always cover their cash expenditures, and they can choose to raise capital by issuing equity. A share (also referred to as an equity share) of stock entitles the holder to a part of ownership in a corporation. To compensate stockholders for not receiving interest that they might have received with other investments, companies usually pay them dividends. Dividends can vary over time depending on the company's performance and can also be viewed as a part of the company's profit redistributed to its owners. Therefore, the price of a stock normally drops by approximately the value of the dividend at the ex-div date, which is the last date after which the buyer of a stock is not entitled to receive the next dividend payment. Note that dividends can be expressed as discrete dividends or as a continuous equivalent dividend yield q.

When buying stocks, investors typically expect the stock price to increase in order to make profit from their investment. On the other hand, consider an investor who believes a stock price is going to decrease over time. She is then interested in having a short position in this stock. If her portfolio doesn't contain it, she can enter into a repurchase agreement or *repo*. This is

a transaction in which the investor borrows the stock from a counterparty that holds the stock and agrees to give it back at a specific date in the future. Repos allow the investor to hold the stock and sell it short immediately in the belief that she can buy it back later in the market at a cheaper price and return it to the lending counterparty. Repos play a large role as speculative instruments. It is interesting to note that stock lenders are, for the most part, people who are just not planning to trade in it. They could be investors that own the stock in order to take control of the company, and repos offer them the advantage to earn an added income paid by the borrowers. The rate of interest used is called the *repo rate* or *borrowing cost*.

The stock price's behaviour is not the only important parameter that should be taken into account when trading stocks. An investor should be cautious with liquidity that can be quantified by looking at the average daily traded volume. A stock is said to have liquidity if there are many active participants buying and selling it, and that one can trade the stock at a relatively small bid–ask spread. For a stock to be considered liquid, one should be able to buy or sell it without moving its price in the market. Take the scenario where an investor wants to sell a large position in stocks. If the stock is not liquid enough, it is likely that the investor wouldn't find a buyer at the right time and would not be able to make a profit from his investment. At least, it is possible that the seller might not find a buyer who is willing to buy the stock at its fair price, and would have to sell at a price below the actual price just to conduct the transaction. Note that liquidity is correlated to the stock price. If the latter is too high or too low, the liquidity of the stock suffers. Expensive stocks are not affordable to all investors, causing the traded volume to be low. Alternatively, very cheap stocks may be de-listed.

Another parameter that has to be taken into account is corporate actions. These constitute an event initiated by a public company, and that may have a direct or indirect financial impact on the security. Companies can choose to use corporate actions to return profits to shareholders (through dividends for example), to influence the share price or for corporate restructuring purposes. Stock splits and reverse stock splits are respectively used to increase and decrease the number of outstanding shares. The share price is then adjusted so that market capitalization (the share price times the number of shares outstanding) remains the same. These events can be an interesting solution to increase the liquidity of a stock. Finally, mergers are an example of corporate actions where two companies come together to increase their profitability. From a trading perspective, one should be cautious with corporate actions since they can have a great impact on the price or the liquidity of a stock.

Let us now analyse the *forward* price of a stock, which is defined as the fair value of the stock at a specific point of time in the future. The forward price of a stock can be viewed as equal to the spot price plus the cost of carrying it. Consider a share that pays no dividends and is worth \$50. Assume that the 6-month interest rates are equal to 6%. Here, the cost of carry is equal to the interest that might be received by the stockholder if he had immediately sold his shares and invested his money in a risk-free investment. This represents a cost for the stockholder that will be reflected in a higher forward price. Therefore, the 6-month forward price of the stock would be equal to $50\mathrm{e}^{6\%\times 6/12} = \$51.52$.

If a stock provides an additional income to the stockholder, this causes the cost of carry to decrease, since the stock also becomes a source of profit. Dividends and stock loans constitute a source of income when carrying a stock. Therefore, those parameters decrease the forward price whereas interest rates increase it. Let r, q and b respectively denote the risk-free rate, the dividend yield and the repo rate for a period T. Then the forward price $F_0(T)$ for a specific stock S is given as follows: $F_0(T) = S_0 \times \mathrm{e}^{(r-q-b)\times T}$. From this relationship we can see that

an increase of 1% in the stock price will result in a 1% increase in the forward price, all else being equal.

1.3.2 Foreign Exchange

A currency is a financial instrument that can be traded in terms of spot or forward contracts in foreign exchange markets. Most of the major currencies are very liquid and can involve large transactions. However, one should be cautious with exchange rate quotes and be clear on the foreign exchange (FX) market's conventions. FX futures are always quoted in number of US dollars (USD) per one unit of foreign currency. Spots and forward prices are quoted in the same way; for the British pound GBP, the euro EUR, the Australian dollar AUD and the New Zealand dollar NZD, the spot and forward quotes show the number of USD per one unit of foreign currency. These quotes can be directly compared to futures quotes. For all other major currencies, forward and spot prices are quoted in number of units of foreign currency per one USD. For instance, if the spot exchange rate between GBP and USD is equal to 2, this means 1 GBP = 2 USD.

A foreign currency entitles the holder to invest it at the foreign risk-free interest rate r_f. If an investor converts the FX into domestic currency, he can make a deposit at the domestic risk-free rate r_d. A currency can then be viewed as a stock with a dividend yield equal to r_f. Let S_0 denote the current spot price expressed in dollars of one unit of a foreign currency and $F_0(T)$ denote the fair value of the forward price at time T expressed in dollars of one unit of a foreign currency:

$$F_0(T) = S_0 \times \mathrm{e}^{(r_d - r_f)\times T}$$

The market forward price can be different from the fair value of the forward price expressed above. This event leads to an *arbitrage* opportunity, which is an opportunity to make a profit without bearing risks.

Finally, if a trader wants to exchange a currency A for a currency B but cannot find a quoted price for the exchange rate, he can use the available exchange rates of these currencies with respect to a reference currency C. He would then compute the cross rate A/B as follows:

$$A/B = A/C \times C/B$$

Foreign exchange is discussed in more detail in the pre-hybrid derivative asset class analysis of Chapter 18.

1.3.3 Indices

A stock market index is composed of a basket of stocks and provides a way to measure a specific sector's performance. Stock market indices can give an overall idea about the state of an economy, as is the case for broad-base indices that include a broad set of equities that represent the performance of a whole stock market. These indices are the most regularly quoted and are composed of large-cap stocks of a specific stock exchange, such as the American S&P 500, the Japanese Nikkei, the German DAX, the British FTSE 100, the Hong Kong Hang Seng Index and the EuroStoxx 50. A stock market index can also be thematic or can cover a specific sector such as the technology or banking sectors.

An index value can be computed in two ways. For price-weighted indices, such as the Dow Jones Industrial Average in the US, each component's weight depends only on the price of the

stocks and does not take into account the size of the companies. Therefore, a price-weighted index value is sensitive to price movements even if it only affects one of its constituent stocks. Another way to compute an index is based on the market capitalization of stocks. This is the case of market-value-weighted indices, also called capitalization-weighted indices, where the largest companies have the greatest influence on their price. The Eurostoxx 50 index and the Hang Seng are good examples of capitalization-weighted indices.

1.3.4 Exchange-traded Funds

Much like stocks, an *exchange-traded fund* (or ETF) is an investment vehicle that is traded on stock exchanges. An ETF holds assets such as stocks or bonds and is supposed to trade at (at least approximately) the same price as the net asset value of its assets – throughout the course of the trading day. Since diversification reduces risk, many investors are interested in indices or baskets of assets; however, it is impractical to buy indices because of the large numbers of constituent stocks and the need to rebalance with the index. Therefore, ETFs can be a great solution since one can often find ETFs that track a specific index, such as the Dow Jones Industrial Average or the S&P 500. In one transaction the investor gains exposure to the whole index without having to buy all the stocks composing the index and adjust their weights as the index's weights are changed.

ETFs generally provide transparency as well as the easy diversification across an entire index. They can have low costs and expense ratios when they are not actively managed and typically have lower marketing, distribution and accounting expenses. Another advantage of ETFs is the tax efficiency of index funds, while still maintaining all the features of ordinary stocks, such as limit orders, short selling and options. For an investor, one disadvantage can be that in some cases, and depending on the nature of the ETF and the complexities involved in its management, relatively significant fees may be charged. Because ETFs can be traded like stocks, some investors buy ETF shares as a long-term investment for asset allocation purposes, while other investors trade ETF shares frequently to implement investment strategies. ETFs and options on ETFs can also serve as hedging vehicles for some derivatives.

1.3.5 Forward Contracts

A forward contract is an agreement between two parties to buy or sell an asset at a specified point of time in the future. This is a pure over-the-counter (OTC) contract since its details are settled privately between the two counterparties. When issuing a forward contract, the price agreed to buy the asset at maturity is called the strike price. Trading in forwards can be for speculative purposes: (1) the buyer believes the price of the asset will increase from the trade date until the maturity date; (2) the seller thinks the value of the asset will appreciate and enters into a forward agreement to avoid this scenario. Additionally, forward contracts can serve as hedging instruments.

Generally, the strike price is equal to the fair value of the forward price at the issue date. This implies that forward contracts are usually arranged to have zero mark-to-market value at inception, although they may be off-market. Examples include forward foreign exchange contracts in which one party is obligated to buy foreign exchange from another party at a fixed rate for delivery on a preset date. In order to price a forward contract on a single asset, one should discount the difference between the forward price and the strike price. Assuming that $F_t(T)$ is the theoretical forward price of the asset, the value at time t of the forward contract

$\text{Forward}_t(T)$ is computed as follows:

$$\text{Forward}_t(T) = (F_t(T) - K) \times \mathrm{e}^{-r\times(T-t)}$$

The main advantage of forwards is that they offer a high degree of flexibility to both parties involved, allowing them to set any contract specifications as long as they are mutually accepted. This is due to the fact that forward contracts trade in OTC markets and are not standardized contracts. Besides, it is important to note that a forward contract is an obligation and not an option to buy/sell the asset at maturity. However, the risk remains that one party does not meet its obligations and can default. This risk, called the counterparty risk, is the main disadvantage encountered in trading forwards.

Exercise

Suppose that John believes the stock price of Vodafone will appreciate consistently over the course of a year. Assume that Vodafone is worth £80 and the 1-year LIBOR rate r is equal to 6%. Also, the dividend yield q is equal to 2% and the borrowing costs are null. John decides to enter into a 1-year forward contract allowing him to buy 1,000 shares of Vodafone in one year at a strike price of £82. After one year, Vodafone's spot price is equal to £86. Did John realize a profit from this transaction?

Discussion

First of all it is interesting to compute the theoretical value of the 1-year forward price F_0 of Vodafone that is given by $F_0 = 80 \times \mathrm{e}^{(6\%-2\%)\times 1} = £83.30$. As the theoretical forward price is higher than the strike price K, John has to pay a premium $\text{Forward}_{\text{price}}$ for the forward contract that is equal to the number of shares times the present value of the difference between the forward price and the strike price, as follows:

$$\begin{aligned}\text{Forward}_{\text{price}} &= 1,000 \times (F_0 - K) \times \mathrm{e}^{-rT} \\ &= 1,000 \times (83.30 - 82) \times \mathrm{e}^{-5\%\times 1} = £1,224\end{aligned}$$

At the end of the year, the forward contract entitles John to receive 1,000 shares of Vodafone at £82 with a market value equal to £86. Therefore, John makes a profit equal to $1,000 \times (86 - 82) = £4,000$ knowing that he paid £1,224 as a forward contract premium.

1.3.6 Futures

A futures contract is an exchange-traded contract in which the holder has the obligation to buy an asset on a future date, referred to as the final settlement date, at a market-determined price called the futures price. The price of the asset on the final settlement date is called the settlement price. The contract specifications, including the quantity and quality of the asset as well as the time and place of delivery, are determined by the relevant exchange. The asset is most often a commodity, a stock or an index. Stock market index futures are popular because they can be used for hedging against an existing equity position, or speculating on future movements of the asset.

Futures constitute a safer investment since the counterparty risk is (almost) totally eliminated. Indeed, the clearing house acts as a central counterparty between the buyer and the seller

and also provides a mechanism of settlement based on *margin calls*. Futures are marked-to-market (MTM) on a daily basis to the new futures price. This rebalancing mechanism forces the holders to update daily to an equivalent forward purchased that day. On the other hand, the benefits of having such standardized contracts are slightly offset by the lack of flexibility that one has when setting the terms of an OTC forward contract. The futures contract is marked-to-market on a daily basis, and if the margin paid to the exchange drops below the margin maintenance required by the exchange, then a margin call will be issued and a payment made to keep the account at the required level. Margin payments offset some of the exchange's risk to a customer's default.

The quoted price of a futures contract is the futures price itself. The fair value of a future is equal to the cash price of the asset (the spot value of the asset) plus the costs of carry (the cost of holding the asset until the delivery date minus any income). When computing the fair value of futures on commodity, one should take into account the interest rates as well as storage and insurance fees to estimate the costs of carry.

As long as the deliverable asset is not in short supply, one may apply arbitrage arguments to determine the price of a future. When a futures contract trades above its fair value, a cash and carry arbitrage opportunity arises. The arbitrageur would immediately buy the asset at the spot price to hold it until the settlement date, and at the same time sell the future at the market's futures price. At the delivery date, he would have made a profit equal to the difference between the market's futures price and the theoretical fair value. Alternatively, a reverse cash and carry arbitrage opportunity occurs when the future is trading below its fair value. In this case, the arbitrageur makes a risk-free profit by short-selling the asset at the spot price and taking at the same time a long position in a futures contract at the market's futures price. When the deliverable asset is not in plentiful supply, or has not yet been created (a corn harvest for example), the price of a future is determined by the instantaneous equilibrium between supply and demand for the asset in the future among the market participants who are buying and selling such contracts. The convenience yield is the adjustment to the cost of carry in the non-arbitrage pricing formula for a forward and it accounts for the fact that actually taking physical delivery of the asset is favourable for some investors. These concepts are discussed at length for the various asset classes in Chapters 17 and 18 where futures and forward curves are analysed.

1.4 SWAPS

1.4.1 Interest Rate Swaps

Interest rate swaps (IRSs) are OTC agreements between two counterparties to exchange or swap cashflows in the future. A specific example of an IRS is a plain vanilla swap, in which two parties swap a fixed rate of interest and a floating rate. Most of the time, LIBOR is the floating interest rate used in a swap agreement. In an IRS, the notional is the principal amount that is used to compute interest percentages, but this sum will not actually change hands. Payments are netted, because all cashflows are in the same currency; for instance payment of 5% fixed and receipt of 4% floating will result in a net 1% payment. Payments are based on the floating interest rate observed at the start of the period, but not paid until the end of the period. More exotic swaps exist where cashflows are in different currencies, examples of which can be found below.

The *payer* on the swap is the person who agrees to pay the fixed rate (and receive the floating rate) on a vanilla swap. The payer is concerned that interest rates will rise and would then be referred to as long the swap. The *receiver* is the person who agrees to receive the fixed rate (and pay the floating rate) on an IRS. The receiver expects interest rates to fall and would therefore be referred to as being short the swap. It is because of the different methods of borrowing that interest rate swaps are useful. A company may either borrow money at fixed or variable rates; it would borrow fixed if it thought rates were going up and variable if it thought they were going to fall. An IRS will allow the company to change borrowing styles part way through the term of the original loan. These are OTC products and, as such, can be tailored to an investor's cashflow needs accordingly.

Consider for example a 5-year 3-month borrowing facility. The 5 years are split into 3-month periods; at the beginning of each period the 3-month LIBOR rate is set and applied to the loan. At the end of each period (the reset date), the interest is paid, and a new LIBOR rate is set for the next 3-month period. A company with such a facility may approach another institution and arrange an IRS. The institution would agree to pay LIBOR to the company at the end of each 3-month period in exchange for interest payments from the company at a fixed rate.

A basis swap is a particular type of IRS where a floating rate is swapped for a different floating rate. These transactions are used to change the floating rate basis from one index to another, e.g. exchanging 3-month LIBOR for 6-month LIBOR, or 3-month T-bill rate for 6-month Fed Funds. The floating indices used in these swaps range from LIBOR rates of different tenors or possibly different currencies, to other floating rates.

To compute the value of a swap, one should calculate the net present value (NPV) of all future cashflows, which is equal to the present value from the receiving leg minus the present value from the paying leg. Initially, the terms of a swap contract are defined in such a way that its value is null, meaning that one can enter into the swap at zero cost. In the case of an IRS, the fixed rate is agreed such that the present value of the expected future floating rate payments is equal to the present value of future fixed rate payments.

Exercise

Let E denote the 3-month EURIBOR rate. Consider an interest rate swap contract where Party A pays E to Party B, and Party B pays $24\% - 3 \times E$ to Party A. Let N denote the notional of this swap. Can you express this deal in simpler terms?

Discussion

Party A pays E and receives $24\% - 3 \times E$. This means that Party A receives $24\% - 4 \times E = 4 \times (8\% - E)$. This contract is then equivalent to an interest rate swap arrangement where Party A (the receiver) receives 8% from Party B (the payer), and pays E to Party B. The notional of the equivalent contract is equal to $4 \times N$.

1.4.2 Cross-currency Swaps

A currency swap is another popular type of swap in which cashflows are based on different currencies. Unlike an IRS, in a currency swap the notional principal should be specified in both currencies involved in the agreement. Here, a notional actually changes hands at

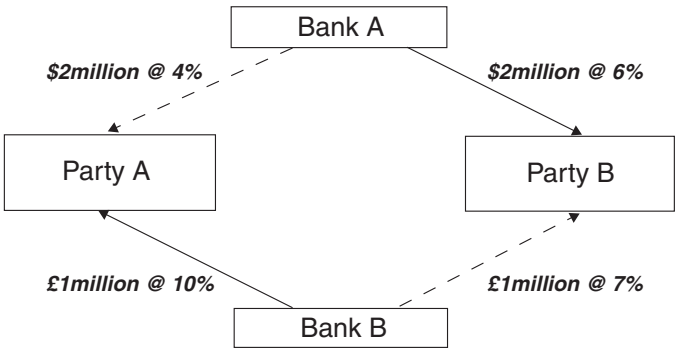

Figure 1.1 Borrowing rates.

the beginning and at the termination of the swap. Interest payments are also made without netting. It is important to note that principal payments are usually initially exchanged using the exchange rate at the start of the swap. Therefore, notional values exchanged at maturity can be quite different. Let's consider an example of a fixed-for-fixed currency swap, where interest payments in both currencies are fixed, to clarify the payoff mechanism and the cross-currency swap's use in transforming loans and assets.

Figure 1.1 shows the case of an American company (Party A) that wants to raise £1m from a British bank (Bank B) and a British company (Party B) that needs to borrow $2m from an American bank (Bank A). In this example, we assume that 1 GBP = 2 USD. Let's keep in mind that interest rate values depend on the creditworthiness of the borrower. In this example, both companies have similar credit ratings but banks tend to feel more confident when lending to a local company. Bank B is then ready to lend £1m to Party A at a fixed rate of 10% per annum over a 3-year period, whereas the interest rate is fixed at 7% for Party B. For the same reasons, Bank A accepts to lend its funds at a fixed rate of 4% for Party A, whereas the interest rate would be equal to 6% for Party B.

Both companies decide to enter into a currency swap agreement, described in Figure 1.2, to benefit from the difference of loan rates. Party A borrows $2m from Bank A at 4% annual fixed rate and Party B borrows £1m from Bank B at a 7% annual rate. At the start date of the swap, both principals are exchanged, which means that Party A gives $2m to Party B and

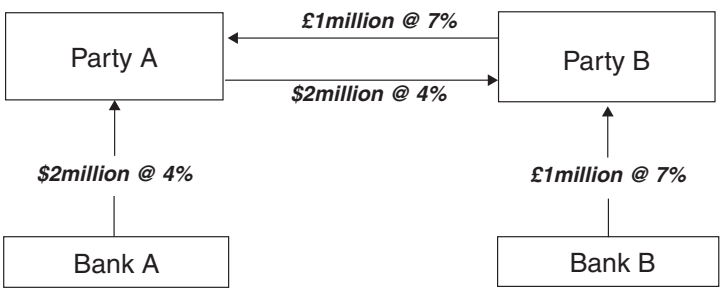

Figure 1.2 Currency swap (fixed for fixed).

receives £1m. At the end of each year, Party A receives $80,000 from Party B (used to pay the 4% interest to Bank A) and pays £70,000 to Party B (used to pay the 7% interest to Bank B). At the outset of the swap, the notional amounts are exchanged again to reimburse the banks. The overall effect of this transaction is that both companies raised funds at lower interest rates. Party A has borrowed £1m at a rate of 7% instead of 10%. Party B has also made a profit from this currency swap since it has paid 4% interest rate instead of 6%. Note that this is a fixed-for-fixed currency swap. It is also possible to swap fixed-for-floating.

1.4.3 Total Return Swaps

A total return swap is a swap agreement in which a party pays fixed or floating interest and receives the *total return* of an asset. The total return is defined as the capital gain or loss from the asset in addition to any interest or dividends received during the life of the swap. Note that the party that pays fixed or floating rates believes the asset's value will appreciate. This party receives the positive performance of the asset and pays its negative performance. A total return swap enables both parties to gain exposure to a specific asset without having to pay additional costs for holding it.

An equity swap is a particular type of total return swap where the asset can be an individual stock, a stock index or a basket of stocks. The swap would work as follows: if an investor believes a specific share will increase over a certain period of time, she can enter into an equity swap agreement. Obviously, this is a purely speculative financial instrument since the investor does not have voting or any other stockholder rights. Compared to holding the stock, she does not have to pay anything up front. Instead, she would deposit an amount of money, equal to the spot price of the stock (a different amount in the case of a margin), and would receive interest on it.

Thus, the investor creates a synthetic equity fund by making a deposit and being long the equity swap. Typically, equity swaps are entered into to gain exposure to an equity without paying additional transaction costs, locally based dividend taxes. It also enables investors to avoid limitations on leverage and to get around the restrictions concerning the types of investment an institution can hold.

1.4.4 Asset Swaps

An asset swap is an OTC agreement in which the payments of one of the legs are funded by a specified asset. This asset can be a bond, for example, where the coupons are used as payments on one leg of the swap, but the bond, and generally the asset underlying this swap, does not exchange hands. This allows for an investor to pay or receive tailored cashflows that would otherwise not be available in the market.

1.4.5 Dividend Swaps

Lastly, a dividend swap is an OTC derivative on an index or a stock and involves two counterparties who exchange cashflows based on the dividends paid by the index or the stock. In the first of the two legs a fixed payment is made (long the swap), and in the second leg the actual dividends of the index or the stock are paid (short the swap). The fixed leg payments involve a fixed amount that depends on the initial price of the index of the stock. The cashflows are exchanged at specified valuation periods and are based upon an agreed notional amount. In the

case of an index dividend swap, or a dividend swap on a basket of stocks, the dividends of the constituents are weighted by the same weights of the index/basket constituents. The dividend swap is a simple and price effective tool for investors to speculate on future dividends directly, and it can also serve as a vehicle for traders holding portfolios of stocks to hedge dividend risk. The liquidity of such swaps has increased in recent years for both these reasons.